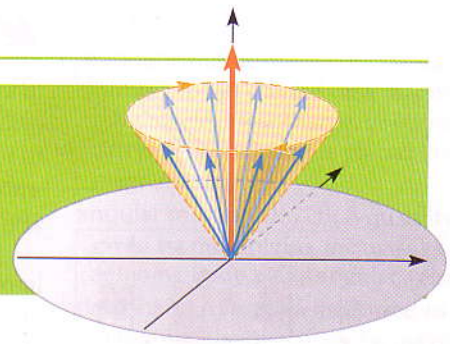
Chapter 3

Basic Principles of MR Signal Generation



All magnetic resonance imaging, including fMRI, relies on a core set of physical principles that were discovered by Rabi, Bloch, Purcell, and other pioneers during the first half of the twentieth century. These principles are elegant in their simplicity. They begin with the properties of single atomic nuclei and progress, step-by-step, to the signal measured using MRI. Yet, they are also rigorous and quantitative.

The greatest challenge faced by teachers of MRI is to do justice to both the elegance and the rigor of signal generation. Some students prefer to learn by intuition and analogy, while others are most comfortable when working through the underlying equations. To accommodate both groups while maintaining completeness of scope, we will discuss the principles of signal generation in two sections of the chapter, which follow independent but corresponding paths through the material (Figure 3.1). We begin the chapter with a *conceptual path* that uses textual descriptions and analogies to illustrate and reinforce the key concepts of signal generation. This is followed by the *quantitative path*, which covers the same basic principles using equations and mathematical notation, allowing interested students to understand the exact contributions of different components to the measured MR signal.

Both paths cover the same topics in the same order; so either one can provide the background necessary for reading subsequent chapters in the book. Thus, instructors and students can choose to either take one path or go back and forth between them, using the concepts to simplify the equations, and the equations to elaborate and give more specific meaning to the key concepts.

Conceptual Path

To provide students with an intuitive understanding of key concepts like spin, precession, and relaxation, this overview emphasizes the underlying principles rather than the mathematical formulas. Although individual concepts might seem complex, they build on each other in a step-by-step fashion. First we consider individual spins (i.e., atomic nuclei), then we discuss how those spins are influenced by magnetic fields and the delivery of electromagnetic energy. We end by describing how spins subsequently release energy over time to generate the signal measured by MRI.

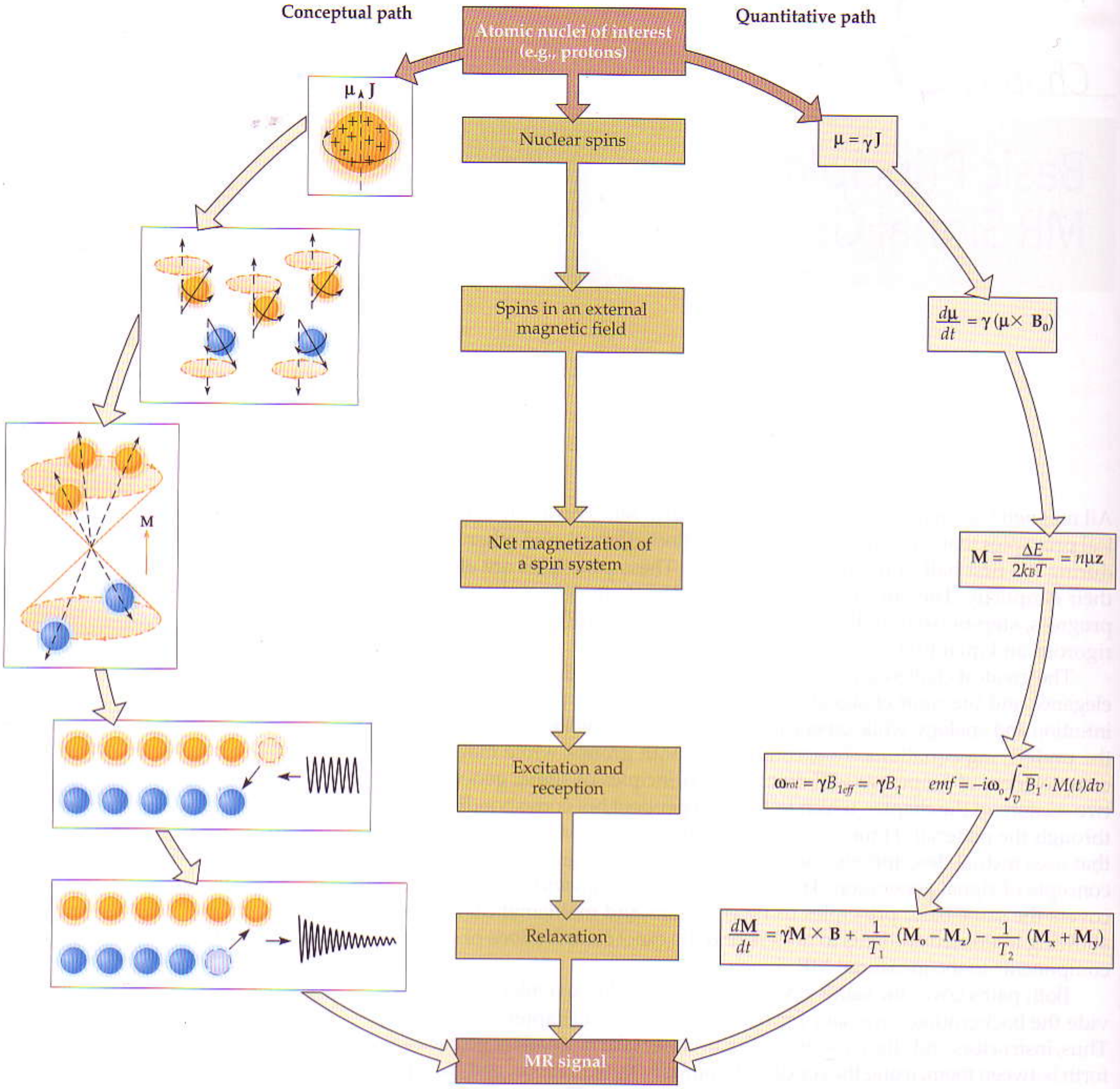


Figure 3.1 Overview of the chapter. We have structured this chapter (and Chapter 4) along two parallel paths, each covering the basic principles of MR signal generation: from proton spins to magnetic resonance to excitation and reception. The initial *conceptual path* uses physical models and analogies to cover these principles in a straightforward and intuitive manner, with the minimally necessary jargon, and using no equations. Then, for the benefit of technically curious readers, the subsequent *quantitative path* moves systematically through the equations that govern the generation and reception of the MR signal. Although the quantitative path contains many equations, we have worked to make those equations as accessible as possible, by defining terms and labeling quantities throughout.

Nuclear Spins

All matter is composed of atoms, which contain three types of particles: protons, neutrons, and electrons. The protons and neutrons are bound together in the atomic nucleus. Different atoms have different nuclear compositions; hydrogen nuclei, which are by far the most abundant in the human body, consist of single protons. Because of their abundance, hydrogen atoms are the most commonly-imaged nuclei in MRI, and we will focus on the properties of single protons throughout this discussion.

Consider a single proton of hydrogen. Under normal conditions, thermal energy causes the proton to spin about its axis (Figure 3.2A). This spin motion has two effects. First, because the proton carries a positive charge, its spin generates an electrical current on its surface, just as a moving electrical charge in a looped wire generates current. This current on the surface of the proton creates a small magnetic source and a torque when it is placed within a magnetic field. The strength of this magnetic source, or the maximum torque (i.e., the torque generated when an external magnetic field is at a right angle to the axis of the proton's spin) per unit of magnetic field strength is called the magnetic moment (μ). Second, because the proton has an odd-numbered atomic mass (i.e., a mass of one), its spin results in an angular momentum, or J. Both μ and J are vectors pointing in the same direction, given by the right-hand rule, along the spin axis. To remember the difference between the magnetic moment and angular momentum, think of the proton as a spinning bar magnet. As the magnet spins, the changing magnetic field generates a magnetic moment and the moving mass results in angular momentum (Figure 3.2B).

For a nucleus to be useful for MRI, it must have both a magnetic moment and an angular momentum. If both are present, the nucleus is said to possess the nuclear magnetic resonance (NMR) property. A few nuclei that are used often in NMR include ^{1}H, ^{13}C, ^{19}F, ^{23}Na, and ^{31}P. A nucleus with the NMR property can be referred to as a spin and a collection of such nuclei at one spatial location is known as a spin system. If a nucleus does not have both characteristics, it cannot be studied using magnetic resonance. For example, in a nucleus that has an even number of protons and an even number of neutrons, its magnetic moment can be cancelled by distributing the same amount of charges in opposite directions, and thus it would be invisible to magnetic resonance imaging.

An average person who weighs 150 pounds contains approximately 5×10^{27} hydrogen protons (because of the high water content in our bodies) and much smaller numbers of other

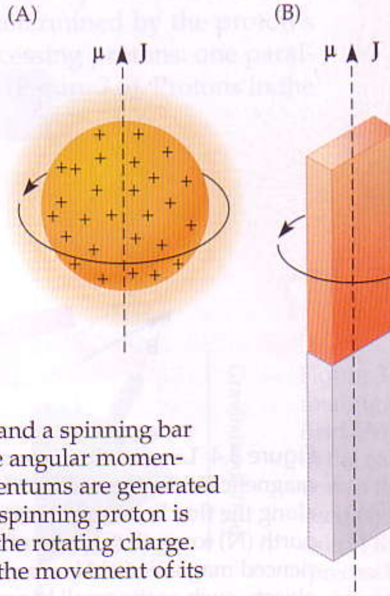


Figure 3.2 Similarities between a spinning proton (A) and a spinning bar magnet (B; north and south poles indicated). Both have angular momentums (J) and magnetic moments (μ). The angular momentums are generated by the spinning masses. The magnetic moment for the spinning proton is generated by the electric current, which is induced by the rotating charge. The magnetic moment for the bar magnet comes from the movement of its internal magnetic field.

magnetic moment (μ) The torque (i.e., turning force) exerted on a magnet, moving electrical charge, or current-carrying coil when it is placed in a magnetic field.

angular momentum (J) A quantity given by multiplying the mass of a spinning body by its angular velocity.

right-hand rule A method used to determine the direction of a magnetic moment generated by a moving charge or electrical current. If the fingers of the right hand are curled around the direction of spin, then the magnetic moment will be in the direction indicated by the thumb.

NMR property A label for atomic nuclei that have both a magnetic moment and angular momentum, which together allow them to exhibit nuclear magnetic resonance effects.

spins Atomic nuclei that possess the NMR property; that is, they have both a magnetic moment and angular momentum.

spin system A collection of atomic nuclei that possess the NMR property within a spatial location.

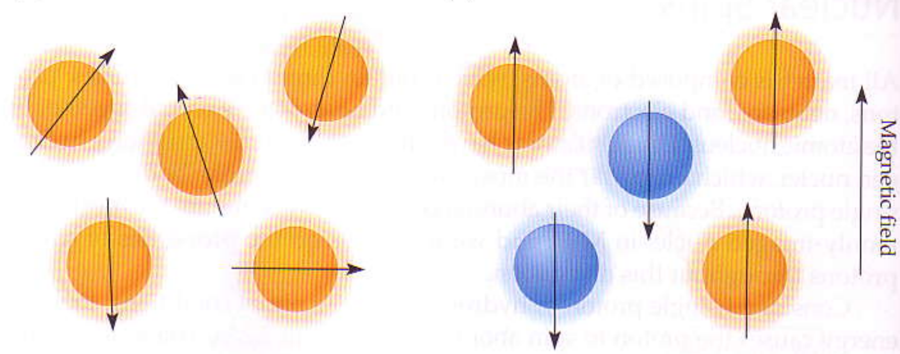


Figure 3.3 Magnetic fields cause the alignment of nuclei that have the NMR property. (A) In the absence of an external magnetic field, protons in free space will have their spin axes aligned randomly. (B) When an external magnetic field is introduced, each proton's axis of spin will tend to take one of two states: either aligned along (parallel to) or against (antiparallel to) the magnetic field. More of the spins will enter the parallel state, resulting in a net magnetization that is parallel to the scanner's magnetic field.

net magnetization (M) The sum of the magnetic moments of all spins within a spin system.

NMR-property nuclei. Each proton possesses a magnetic moment and angular momentum and is thus a potential contributor to the MR signal. However, in the absence of a strong external magnetic field, the spin axes of the protons are oriented randomly (Figure 3.3A) and tend to cancel each other out. Thus, the sum of all magnetic moments from spins of different orientations, or the **net magnetization (M)**, is infinitesimally small under normal conditions. To increase the net magnetization of the protons, a strong magnetic field must be applied to align the axes of spin of the protons (Figure 3.3B).

Spins in an External Magnetic Field

A classic demonstration of magnetism can be created by sprinkling some iron filings around a standard bar magnet. The filings clump most densely around the poles of the magnet but also form a series of arcs between the poles (Figure 3.4A). These arcs run along the field lines of the magnet and result from the

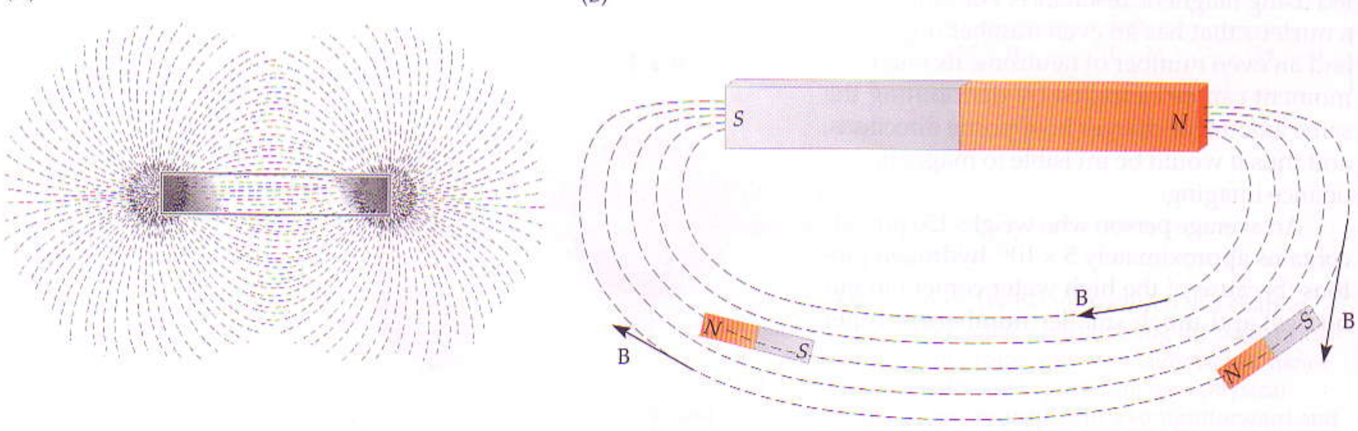


Figure 3.4 Lines of flux in a magnetic field. (A) The alignment of iron shavings in the magnetic field surrounding a bar magnet. (B) A schematic illustration of alignment along the flux lines near a bar magnet. By convention, lines of flux extend from the north (N) to south (S) poles of the magnet. At each point in space, however, the experienced magnetic field is a vector (indicated by the arrows labeled B). Magnetic objects, such as the small bar magnets shown, would align along the flux lines.

tendency of individual iron filings to align with the external field. Figure 3.4B presents a schematic illustration of this alignment, which is driven by the principle of energy minimization. Just as massive objects in a gravitational field tend to lower their energy by falling rather than remaining suspended in midair, magnetically susceptible objects in a magnetic field will orient along the field lines rather than across them. For macroscopic objects like oxygen canisters or iron filings, the alignment process is known as torsion, and it presents safety issues, as discussed in Chapter 2. Note that the magnetic field is still present between the field lines. The pattern of field lines can be interpreted as a mathematical description of the contours of the magnetic field. The density of lines at a particular location indicates the local strength, or flux, of the magnetic field. In magnetic resonance imaging, the main magnetic field of the scanner is often indicated by the symbol B_0.

Protons, like iron filings, change their orientations when placed within an external magnetic field. However, instead of turning to align with the magnetic field, the spinning protons initiate a gyroscopic motion known as precession (Figure 3.5A). Because the precession frequency is determined by the type of nucleus, to a first approximation all protons precess at the same frequency when experiencing the same external magnetic field strength. This characteristic frequency is called the Larmor frequency. To understand precession, imagine a spinning top on a desk (Figure 3.5B). The top does not remain perfectly upright; instead its axis of rotation traces a circle perpendicular to the earth's gravitational field. At any moment in time the top is tilted from the vertical, but it does not fall. Why does the top spin at an angle? A spinning object responds to an applied force by moving its axis in a direction perpendicular to the applied force. A bicycle, for example, is very stable and resists falling over when it is moving at high speeds, due to the gyroscopic effects of its spinning wheels. When a rider leans to one side, the moving bicycle will not fall but will instead turn in that direction. Similarly, a spinning top turns its axis of rotation at an angle perpendicular to the force exerted by gravity, so that the top precesses in a circle around a vertical axis.

The behavior of a proton in a magnetic field is analogous to that of a spinning top in a gravitational field. Specifically, protons precess around an axis parallel to the main magnetic field. The angle between a proton's axis of spin and the direction of the main magnetic field is determined by the proton's angular momentum. There are two states for precessing protons: one parallel to the magnetic field and the other antiparallel (Figure 3.6). Protons in the

flux A measure of the strength of a magnetic field over an area of space.

B_0 The strong static magnetic field generated by an MRI scanner.

precession The gyroscopic motion of a spinning object, in which the axis of the spin itself rotates around a central axis, like a spinning top.

Larmor frequency The resonant frequency of a spin within a magnetic field of a given strength. It defines the frequency of electromagnetic radiation needed during excitation to make spins change to a high-energy state, as well as the frequency emitted by spins when they return to the low-energy state.

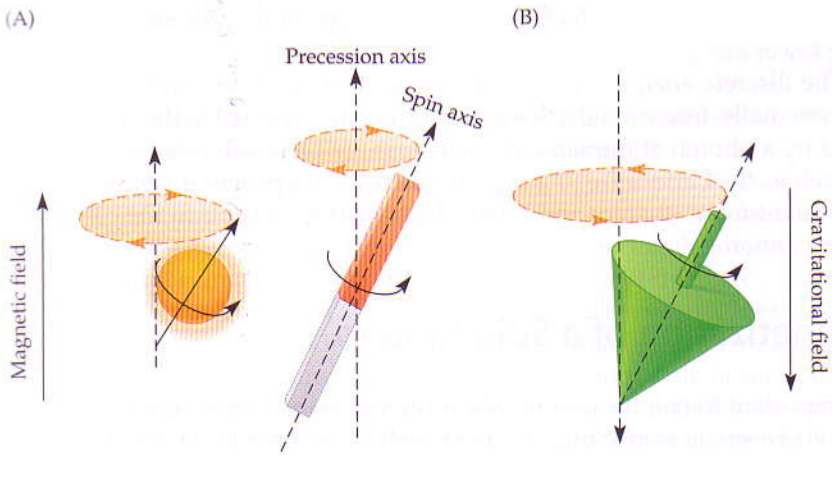


Figure 3.5 Precession. The movement of a rotating proton or magnet within a magnetic field (A) is similar to the movement of a top in the earth's gravitational field (B). In addition to the spinning motion, the axis of spin itself (spin axis) wobbles around the main axis of the magnetic or gravitational field (precession axis). This latter motion is known as precession.

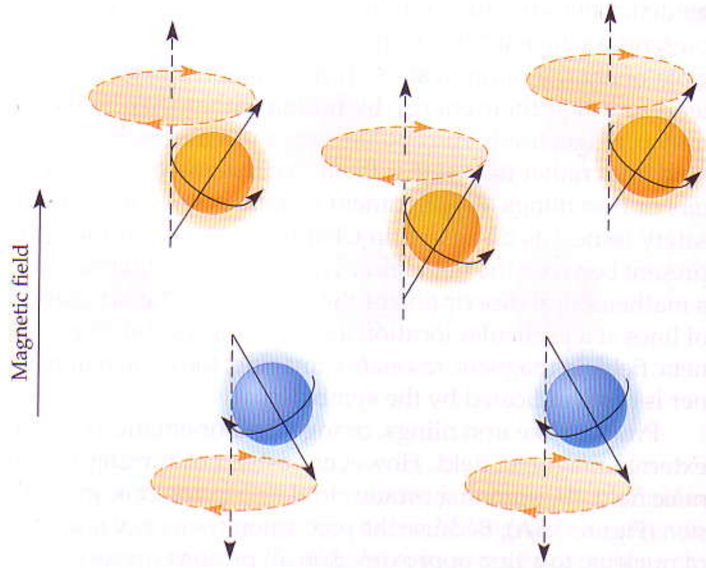


Figure 3.6 High- and low-energy states. Protons in an external magnetic field assume one of two possible states: the parallel state (shown in orange), which has a lower energy level, and the antiparallel state (shown in blue), which has a higher energy level. In the absence of spin excitation, note that there always will be more protons in the parallel state than in the antiparallel state.

parallel state The low-energy state in which an atomic spin precesses around an axis that is parallel to that of the main magnetic field.

antiparallel state The high-energy state in which an atomic spin precesses around an axis that is antiparallel (i.e., opposite) to that of the main magnetic field.

parallel state have a lower energy level than protons in the antiparallel state. The idea of two energy states can be understood by imagining a bar that can rotate around one end (Figure 3.7). There are two vertical positions for the bar: in one it is balanced above the pivot point and in the other it hangs down from the pivot point. The balanced position is a high-energy state and is not very stable; even a small perturbation may tip the bar over and cause it to fall to the hanging position. The only way to keep the bar in a balanced position is to apply an external force that can counteract gravity. That is, energy must be applied to keep the bar in the high-energy state. The hanging position is much more stable, since it is at the minimum energy level for this system. For protons, the parallel (low-energy) state is slightly more stable than the antiparallel state, so there will always be more protons in the parallel state (in the absence of externally delivered energy), with the exact proportions depending on the temperature and the strength of the magnetic field. In Earth's magnetic field at room temperature, roughly equal numbers of protons are in the two energy states, with only slightly more in the parallel state. If the temperature increases, some protons will acquire more energy and jump to the antiparallel state, diminishing but never reversing the already small difference in numbers between the two levels. Conversely, if the temperature decreases, spins will possess less energy and even more protons will remain at the lower energy level.

The discrete energy difference between the two states can be measured experimentally. Interestingly, this energy difference is equal to the energy possessed by a photon at the same frequency as the precession frequency of the proton (i.e., the Larmor frequency). This coincidence provides a basis for unifying quantum mechanics and classical mechanics concepts in the context of spin excitation.

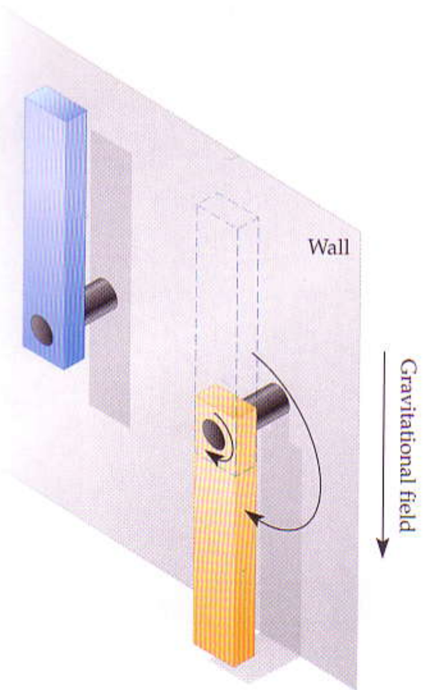


Figure 3.7 High- and low-stability states. Just like a proton in a magnetic field, a bar within the earth's gravitational field can assume two states: the antiparallel state against gravity (blue), which has a higher energy level but is less stable, and the parallel state with gravity, which has a lower energy level but is more stable (orange). Energy must be applied to keep the bar in the high-energy state.

Magnetization of a Spin System

It is important to emphasize that MR techniques do not measure the magnetization of a single atomic nucleus, but instead measure the net magnetization of all nuclei in a volume. We can think of the net magnetization as a vector with two components: a longitudinal component that is either parallel or antiparallel to the magnetic field and a transverse component that is perpendicular to the magnetic field. Because of the enormous number of spins within even the smallest volume, their transverse components will tend to cancel out, and there will be no net magnetization perpendicular to the main magnetic field. The net magnetization **M** is thus a vector whose orientation is along the longitudinal direction and whose magnitude is proportional to the difference between the number of the spins in the parallel and antiparallel states. The more spins at the parallel state, the bigger the **M** (Figure 3.8).

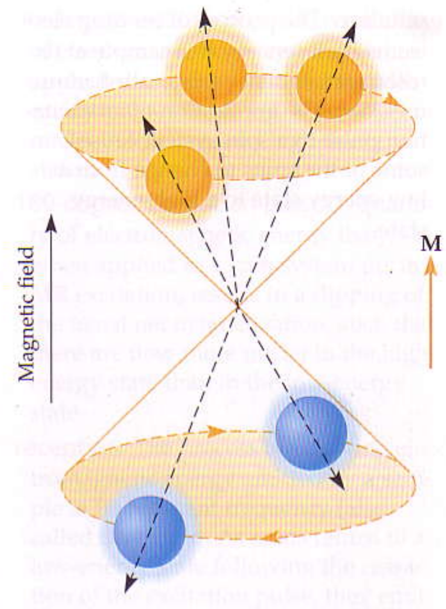


Figure 3.8 Net magnetization. The net magnetization (**M**) is determined by the difference between the number of spins in the parallel state and the number of spins in the antiparallel state. The net magnetization is also called the bulk magnetization.

Because the proportion of parallel spins increases with decreasing temperature, one way to increase the net magnetization is to reduce the temperature. While theoretically possible, this approach is impractical because the temperature would need to be lowered by many degrees to observe a noticeable increase in net magnetization. A more feasible approach, based on the Zeeman effect, is to increase the strength of the external field (Figure 3.9). Just as it would require more energy to lift an object in a stronger gravitational field than in a weaker gravitational field, it takes more energy to shift from a low-energy state to a high-energy state when the external magnetic field is stronger. Therefore, to increase the net magnetization in a sample (i.e., the proportion of parallel spins), one can place that sample in a very strong magnetic field.

Both the increase in net magnetization and the energy difference between the two states are linearly proportional to the strength of the magnetic field. The combination of these two factors means that the theoretical amplitude of the measured MR signal increases quadratically with field strength (e.g., the raw MR signal in a 3-T scanner is four times larger than the signal in a 1.5-T scanner). However, other factors (discussed more extensively in Chapter 8) temper this advantage of high field scanning. Thermal noise in the MR scanner increases approximately linearly with field strength, while some aspects of physiological noise have more complex relationships with field strength. Nevertheless, increasing the strength of the static magnetic field provides significant improvements in the signal-to-noise ratio. For this reason, scanners used for fMRI in humans have increased in field strength, from 1.5 T in the early years, to current standards of 3 T and 4 T. Some scanners used for human fMRI have even stronger magnetic fields of up to 7 T.

longitudinal Parallel to the main magnetic field, or the *z*-direction, of the scanner (i.e., into the bore).

transverse Perpendicular to the main magnetic field of the scanner, in the *x*–*y* plane.

Thought Question

What would happen to the relative proportions of parallel and antiparallel spins in a spin system if we reduced its temperature to near absolute zero?

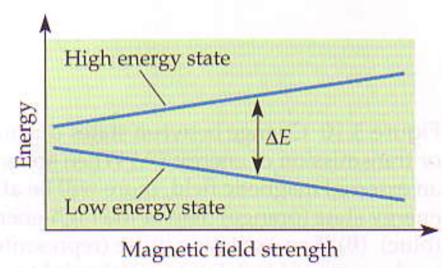


Figure 3.9 The Zeeman effect. The energy difference (ΔE) between the parallel and antiparallel states increases linearly with the strength of the static magnetic field. As the energy difference between the states increases, spins are more likely to remain in the lower-energy state.

The net magnetization of spins within a volume provides the basis for MR signal generation, but net magnetization itself cannot be measured directly under equilibrium conditions. To understand this principle, think of an object whose weight you are trying to estimate. You cannot know the object's weight just by looking at it; instead, you have to lift it. By lifting the object, you perturb its equilibrium state in the gravitational field, and the force observed in reaction to that perturbation allows you to estimate its weight. Measuring the net magnetization of spins in a magnetic field is analogous to measuring the weight of an object; you must perturb the equilibrium state of the spins and then observe how they react to the perturbation. Just as lifting perturbs the position of an object, excitation perturbs the orientation of the net magnetiza-

excitation The process of sending electromagnetic energy to a sample at its resonant frequency (also called transmission). The application of an excitation pulse to a spin system causes some of the spins to change from a low-energy state to a high-energy state.

tion, which allows the precession of the spins within the volume to be visible. Because the net magnetization is the sum of the magnetic moments of all the individual spins, the net magnetization will naturally precess at the same characteristic frequency as that of the individual spins, the Larmor frequency.

Excitation of a Spin System and Signal Reception

Remember that any spin can take either a high-energy state or a low-energy state within a magnetic field (Figure 3.10A). A spin in the low-energy state can jump to the high-energy state by absorbing an amount of energy equivalent to the energy difference between the two states. So transitions between these two states can be triggered by the delivery of energy to the spin system. In MRI, that energy is delivered in the form of radiofrequency pulses. Radiofrequency coils within the MRI scanner bombard spins in the magnetic field with photons, which are actually electromagnetic waves that are adjusted to oscillate at the resonant frequency of the nucleus of interest (e.g., for hydrogen nuclei, the resonant frequency is around 42 MHz per Tesla of static magnet field). The delivery of this energy changes the distribution of spins between the high-energy and low-energy states, with the net effect of favoring transitions from the more abundant state to the less abundant state (i.e., typically, from low to high energy). The process of providing radiofrequency energy to atomic nuclei, so that some spins change from low- to high-energy states, is known as excitation (Figure 3.10B).

If the electromagnetic waves are delivered continuously, an increasing proportion of the spins will have jumped to the higher energy state, eventually reaching a point where there are equal numbers of spins in each state. Once that occurs, there is no net magnetization along the longitudinal direction: the original longitudinal magnetization (and the associated energy) has been fully transferred into the transverse plane. The amount of electromagnetic energy that is exactly sufficient to generate equal numbers of nuclei in each energy

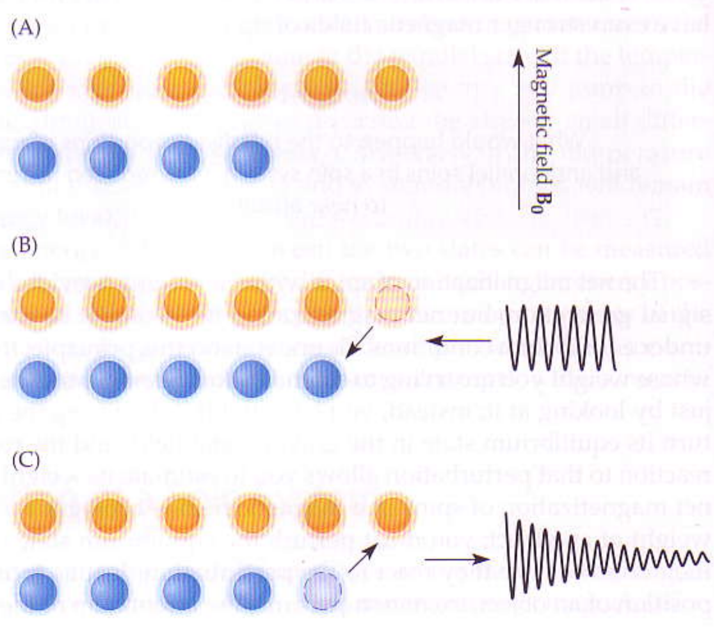


Figure 3.10 Change between states due to absorption or transmission of energy. (A) When spins are placed in an external magnetic field, more will be at the low-energy state (orange) than at the high-energy state (blue). (B) If an excitation pulse (represented in the figure by a wavy black line) with the right amount of energy is applied, some spins will absorb that energy and jump to the high-energy state. (C) After the excitation pulse ceases, some of the spins in the high-energy state will return to the low-energy state, releasing the absorbed energy as a radiofrequency wave with the same frequency as the excitation pulse (i.e., the resonant frequency). The amount of released energy diminishes over time, as indicated by the black line in the figure.

state is known as the 90-degree excitation pulse. This term reflects the classical mechanics perspective of tipping the net magnetization from the longitudinal axis over into the transverse plane, or rotating the vector through 90 degrees. As will be discussed later in this chapter, the measurable MR signal is greatest (and thus imaging is most efficient) when the net magnetization precesses within the transverse plane.

If the electromagnetic waves are left on even longer, beyond the duration needed for a 90-degree excitation pulse, the rate of the spin transitions will slow down as more spins accumulate at the higher energy state. Even so, the number of higher energy spins will continue to grow until the proportions of energy states within the system reach the exact opposites of the original proportions. At this point there are as many high-energy nuclei following excitation as there were low-energy nuclei before excitation. The amount of electromagnetic energy that is exactly sufficient to reverse the numbers of high- and low-energy nuclei, and thus flip the net magnetization vector, is known as the 180-degree excitation pulse. While not typically used to measure fMRI signals, 180-degree excitation pulses are important for increasing contrast on some types of anatomical MR images (see Chapter 5).

Delivering still more electromagnetic energy will cause the spin system to reverse its behavior, such that the proportion of high-energy nuclei will again become lower. If the electromagnetic pulse is left on long enough, enough nuclei will change to the low-energy state that the spin system will return to its original character. Thus, as long as electromagnetic energy is delivered to a spin system, the relative proportions of high- and low-energy spins will change in a predictable, if not always intuitive, manner.

When the electromagnetic waves (radiofrequency pulses) are turned off, the excitation of the atomic nuclei stops. Since excitation has disrupted the thermal equilibrium by creating more high-energy spins than would normally be present, the excess spins at the higher-energy level must return to the lower level (Figure 3.10C), so that equilibrium can be restored. When these high-energy spins fall back to the low-energy state, they emit photons whose energy is equal to the energy difference between the two states (i.e., energy corresponding to the Larmor frequency). Considered from a classical mechanics perspective, the precession of the net magnetization in the transverse plane leads to an electromagnetic oscillation at the Larmor frequency. During this reception period, the changes in transverse magnetization can be detected using a radiofrequency coil tuned to the Larmor frequency. Because the frequencies of excitation and reception are identical (i.e., both at the Larmor frequency), the same radiofrequency coil is often used for both processes. The changing current in these detector coils constitutes the MR signal.

The critical concept underlying excitation and reception is that of the change in net magnetization from the longitudinal axis to the transverse plane. When the net magnetization is along the longitudinal axis, the individual spins' precession cannot be measured in detector coils. But when the net magnetization is tipped into the transverse plane via excitation, its precession can generate an oscillating electric current in reception coils within the scanner.

Relaxation Mechanisms of the MR Signal

The MR signal detected through receiver coils does not remain stable forever. It changes in two ways during signal reception: the transverse magnetization quickly loses coherence, and the longitudinal magnetization slowly recovers.

90-degree excitation pulse A quantity of electromagnetic energy that, when applied to a spin system during MR excitation, results in equal numbers of nuclei in the low- and high-energy states.

180-degree excitation pulse A quantity of electromagnetic energy that, when applied to a spin system during MR excitation, results in a flipping of the usual net magnetization, such that there are now more nuclei in the high-energy state than in the low-energy state.

reception The process of receiving electromagnetic energy emitted by a sample at its resonant frequency (also called detection). As spins return to a low-energy state following the cessation of the excitation pulse, they emit energy that can be measured by a receiver coil.

MR signal The current measured in a detector coil following excitation and reception.

relaxation A change in net magnetization over time.

transverse relaxation (or spin–lattice relaxation) The loss of net magnetization within the transverse plane due to the loss of phase coherence of the spins.

longitudinal relaxation (or spin–spin relaxation) The recovery of the net magnetization along the longitudinal direction as spins return to the parallel state.

T_2 (decay) The time constant that describes the decay of the transverse component of net magnetization due to accumulated phase differences caused by spin–spin interactions.

T_2* (decay) The time constant that describes the decay of the transverse component of net magnetization due to both accumulated phase differences and local magnetic field inhomogeneities. T_2* is always shorter than T_2. BOLD-contrast fMRI relies on T_2* contrast.

T_1 (recovery) The time constant that describes the recovery of the longitudinal component of net magnetization over time.

Together, these changes in the MR signal are called relaxation. The change in transverse magnetization is termed transverse relaxation (or transverse decay), while the longitudinal change is known as longitudinal relaxation (or longitudinal recovery). The parameters governing these two types of relaxation differ across tissues, and this allows a single MRI scanner to collect many types of images.

After spin excitation, the net magnetization is tipped from the longitudinal axis into the transverse plane. Because the net magnetization reflects the vector sum of many individual spins, its amplitude depends on the coherence between those spins, being greatest when they all precess at the same phase and with the same frequency. But over time, the spins lose coherence. Spatially proximate spins may interact like bumper cars on a track, causing some spins to precess at higher frequencies, and some lower. The differences in precession frequencies cause spins to get out of phase with each other, which lead to an exponential decay in the MR signal that is described by a time constant called T_2. Furthermore, any spatial inhomogeneities in the magnetic field will cause different spins to experience different magnetic fields over time, again causing some to precess more rapidly than others. This effect is additive to that of T_2 decay, and the combined effects of spin–spin interactions and magnetic field inhomogeneities is described by the time constant T_2*. As a result of T_2* decay, spins lose coherence relatively quickly (typically within a few tens of milliseconds), resulting in a diminishing net magnetization in the transverse plane (Figure 3.11).

Following excitation, some of the energy of the spin system is emitted as radiofrequency waves that can be detected by receiver coils or antennae as the MR signal. As the spin system loses energy, it recovers back to the same state it was in before the excitation—with the net magnetization aligned along the longitudinal axis (Figure 3.12). The longitudinal recovery is relatively slow, typically on the order of a few hundreds of milliseconds to a few seconds, and is described by the time constant T_1 recovery.

While the T_1 and T_2 relaxations begin simultaneously, their time constants are often very different (T_1 is usually about one order of magnitude larger than T_2) and vary according to the type of tissue that is present. For practical purposes, they can be considered to contribute independently to the MR signal. Depending on when an image is acquired during the relaxation process, either

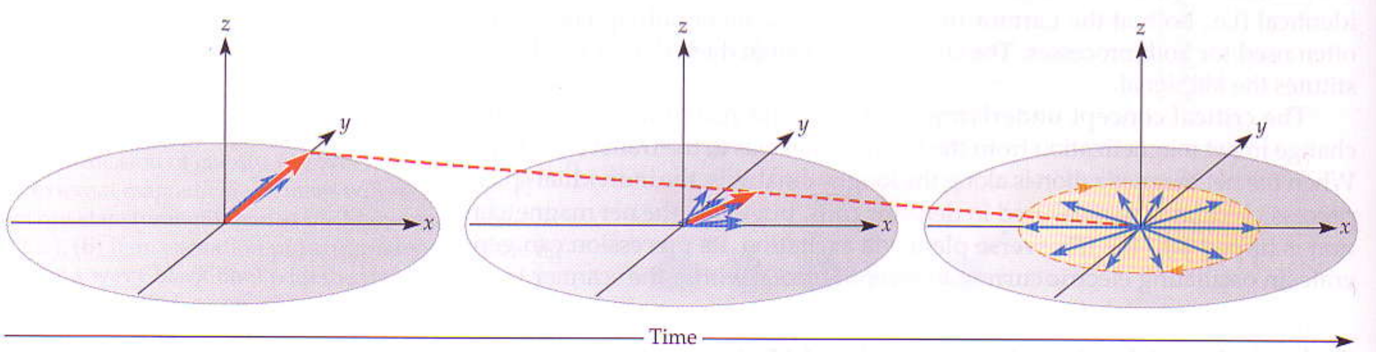


Figure 3.11 A conceptual overview of T_2 decay. After the net magnetization has been tipped into the transverse plane, it rapidly decays because of a loss of coherence among the spins. For most types of tissue, the net magnetization available to generate the MR signal decays to near zero within a few hundred milliseconds (red dashed line).

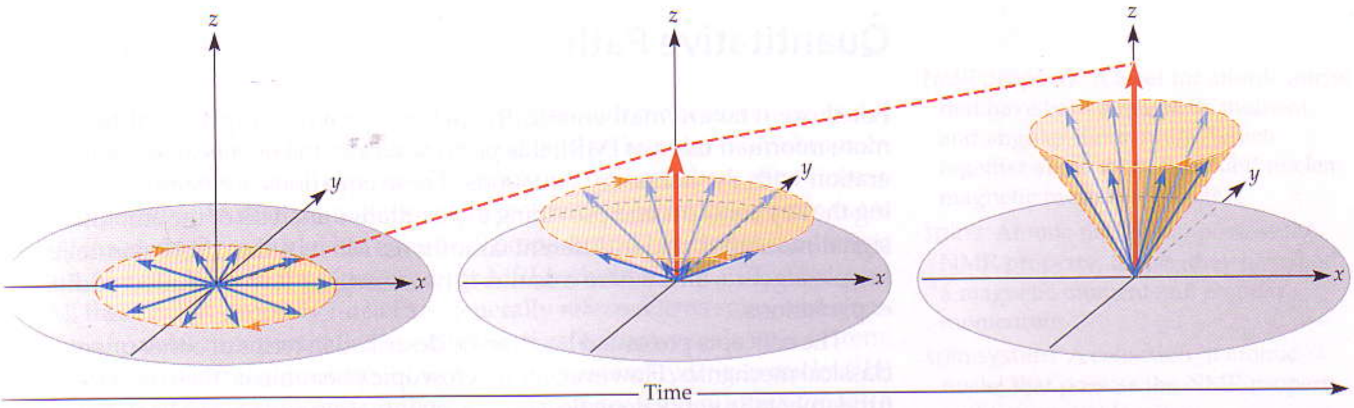


Figure 3.12 A conceptual overview of T_1 recovery. The net magnetization tips into the transverse plane as a result of the absorption of energy by some spins (i.e., those that have changed from low- to high-energy states). Once the excitation pulse ceases, spins begin to release energy back into the surrounding environment. This causes the net magnetization to recover along the longitudinal axis, often returning to near its original amplitude within a few seconds (red dashed line).

T_1 or T_2 (or a combination) will determine the amplitude of the recovered MR signal, and thus the intensity of the image. By choosing appropriate imaging parameters, different tissues (e.g., gray or white matter) will correspond to different intensities, so that investigators can differentiate between them for diagnostic or research purposes. In addition, the speed of T_1 recovery influences the rate at which images can be collected, because T_1 recovery renews the longitudinal magnetization so that it can be excited again.

Conceptual Summary of MR Signal Generation

MR signal generation depends on very simple physical principles. Because of thermal energy, atomic nuclei with the NMR property have an intrinsic characteristic called spin. When placed in a strong magnetic field, these nuclei precess (or wobble) around an axis that is either parallel to the magnetic field (low-energy) or antiparallel to the magnetic field (high-energy). Usually, more nuclei (or spins) take the low-energy state, resulting in a net magnetization parallel to the magnetic field (i.e., longitudinal magnetization). If energy is applied to the nuclei at a particular frequency known as the resonant frequency, some low-energy nuclei will absorb energy from the system and change to the high-energy state, effectively converting the longitudinal magnetization into transverse magnetization. This is known as excitation. Once the energy source is removed, some nuclei will return to the low-energy state by releasing that energy, which restores the longitudinal magnetization. The emitted energy provides the MR signal data that go into our images. The recording of the MR signal is known as reception. The changes in the MR signal over time are known as relaxation, of which there are two types: recovery of the longitudinal magnetization (T_1*) and decay of the transverse magnetization (T_2). By specifying a pulse sequence that targets one of these relaxation parameters, images can be collected that are sensitive to specific properties of the underlying tissue.

Quantitative Path

For those who are mathematically inclined or who simply want to become more informed users of fMRI, this path elaborates the details of MR signal generation with the necessary equations. These equations are helpful for clarifying the key concepts and providing a quantitative analysis of the amount of MR signal measured under different conditions. Most of the discussion relies on simple algebra, and where calculus is necessary we have included additional explanations.

The concepts presented here can be described in terms of either quantum or classical mechanics. However, for macroscopic phenomena, these two views are fundamentally equivalent. Because the quantitative aspects of MR signal generation can be better visualized using concepts from classical mechanics, we will use the notation and descriptions from classical mechanics in this section.

Common Terms and Notations

To ground our description of the MR signal generation process, we first introduce a few common terms and their notations. A scalar is a quantity representing the magnitude of some property. Properties like mass, charge, length, and area are represented by scalars. Scalars may have units; for example, the mass of a person may be 70 kg. Scalars are indicated in italicized type (e.g., M is the amount of net magnetization). A vector is a quantity or phenomenon in which both magnitude and direction are stated. Examples of vectors include force, velocity, momentum, and electromagnetic fields. Vectors are denoted by boldface type (e.g., **M** is the net magnetization vector). Because vectors are directional, the rules for manipulation of vectors are different from those for adding and multiplying scalar numbers.

The dot product, also called the scalar product, of two vectors is a scalar quantity obtained by summing the products of the corresponding components. Consider the two-dimensional vectors **A** and **B**, which can each be represented by the sum of vectors along the x- and y-dimensions. The dot product of A and B is given by $\mathbf{A} \bullet \mathbf{B} = |\mathbf{A}|\ |\mathbf{B}| \cos\theta$, where $|\mathbf{A}|$ and $|\mathbf{B}|$ are the magnitudes of **A** and **B** and θ is the angle between the two vectors when they are placed tail to tail. The dot product may only be performed for pairs of vectors that have the same number of dimensions.

The cross product, or vector product, of two vectors produces a third vector that is perpendicular to the plane in which the first two lie. The value of the cross product of vectors A and B may be defined by $|\mathbf{A} \times \mathbf{B}| = |\mathbf{A}|\ |\mathbf{B}| \sin\theta$, where $|\mathbf{A}|$ and $|\mathbf{B}|$ are the magnitudes of **A** and **B** and θ is the angle between the two vectors. The orientation of the cross product may be determined using the right-hand rule. As one's fingers curl through an angle θ from **A** to **B**, the cross product, or the thumb, points toward the vector perpendicular to the plane defined by **A** and **B.** The magnitude of the cross product is equal to the area of the parallelogram defined by the two vectors. If the components of vectors **A** and **B** are known, then the components of their cross product, $\mathbf{C} = \mathbf{A} \times \mathbf{B}$, may be expressed as:

$$\mathbf{C}_x = \mathbf{A}_y\mathbf{B}_z - \mathbf{A}_z\mathbf{B}_y$$
$$\mathbf{C}_y = \mathbf{A}_z\mathbf{B}_x - \mathbf{A}_x\mathbf{B}_z$$
$$\mathbf{C}_z = \mathbf{A}_x\mathbf{B}_y - \mathbf{A}_y\mathbf{B}_x$$

scalar A quantity that has magnitude but not direction. Scalars are italicized in this text.

vector A quantity with both magnitude and direction. Vectors are in boldface in this text.

dot product The scalar product of two vectors. It is created by summing the products along each dimension.

cross product The vector product of two vectors. Its direction is perpendicular to the plane defined by those vectors and its magnitude is given by multiplying their product times the sine of the angle between them.

Nuclear Spins

Not all nuclei can be used to generate MR signals. For a nucleus to be useful for MRI, it must have both a magnetic moment and an angular momentum. In nuclei with odd numbers of protons (or sometimes odd numbers of neutrons), it is not possible to distribute either the electric charge or the atomic mass evenly. As such, these nuclei have the NMR property and can be detected using MRI. A few nuclei that are often used for MRI include ^{1}H, ^{13}C, ^{19}F, ^{23}Na, and ^{31}P. All these NMR-property nuclei are generally referred to as spins, and a collection of nuclei in a particular spatial location is known as a spin system. Because of the natural abundance of water and therefore hydrogen nuclei (^{1}H) in biological systems, hydrogen is the most commonly used nucleus for MRI. Hence, we will use the hydrogen nucleus, which is a single proton, in the following discussion of spins.

NMR property A label for atomic nuclei that have both a magnetic moment and angular momentum, which together allow them to exhibit nuclear magnetic resonance effects.

spins Atomic nuclei that possess the NMR property; that is, they have both a magnetic moment and angular momentum.

spin system A collection of atomic nuclei that possess the NMR property within a spatial location.

magnetic moment (μ) The torque (i.e., turning force) exerted on a magnet, moving electrical charge, or current-carrying coil when it is placed in a magnetic field.

torque A force that induces rotational motion.

Magnetic Moment

A proton can be visualized as a small sphere with a positive charge distributed over its surface. Because of thermal energy, the proton rotates at a high speed about its axis. The proton's rotation produces a current that in turn generates a small magnetic field, whose strength is known as the magnetic moment and is denoted as μ. Any moving magnet, current-carrying coil, or moving charge has a magnetic moment, which can be quantified as the ratio between the maximum torque on the magnet, coil, or charge exerted by an external magnetic field and the strength of the external field (B). Magnetic moments are measured in "Amperes times meters squared", or Am^2. To provide a visual representation of magnetic moment, we consider a simple rectangular current loop [length (L), width (W), and current level (**I**)] within a magnetic field (Figure 3.13). Note that a spin will trace a circular loop through the magnetic field, so the rectangular loop is just a convenient simplification. The force (**F**) exerted on the segment of wire with length (L) within the magnetic field (**B**) is defined by Equation 3.1a:

$$\mathbf{F} = \mathbf{I} \times \mathbf{B}\, L \tag{3.1a}$$

The maximum force (F_{max}) exerted on the segment of wire occurs when it is perpendicular to the magnetic field, with its quantity given as:

$$F_{max} = IBL \tag{3.1b}$$

Put simply, force is proportional to the strength of the magnetic field (B) and the strength of the current (I). If the magnetic field strength increases, the force will also increase. The effect of this force on objects in the field is to cause them to rotate; the rotational force is known as torque. Thus, torque can be thought of as the change in rotational momentum over time. The maximum torque (τ_{max}) exerted by the magnetic field is given by multiplying the maximum force exerted on the current element by its width. We can account for all wire segments in the current element, or loop, by replacing the length of one segment with the length and width of the loop, which effectively can be replaced by the area (A) of the loop:

$$\tau_{max} = IBLW = IBA \tag{3.2}$$

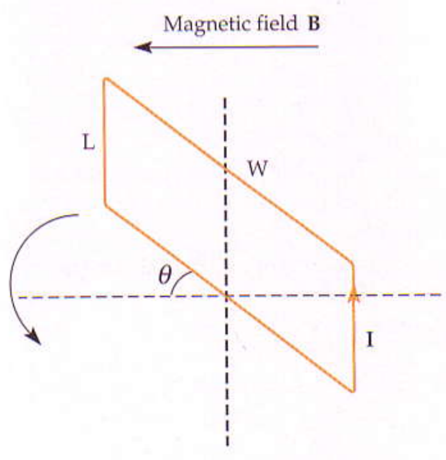


Figure 3.13 Torque on a current loop. A rectangular current loop ($W \times L$) with electric current (I) would experience a torque if placed within a magnetic field B at an angle θ.

Since the magnetic moment $\boldsymbol{\mu}$ is defined as the maximum torque divided by the magnetic field strength (B), its magnitude μ can now be represented as the product of the current and the area of the current loop:

$$\mu = \frac{\tau_{max}}{B} = IA \tag{3.3}$$

Note that the direction of the magnetic moment vector is defined by the right-hand rule based on the flow direction of the current. For a single proton in a strong magnetic field, this direction is generally parallel to the main axis of the magnetic field.

Angular Momentum

Because the proton also has mass, its rotation produces an angular momentum, often denoted as **J**. Angular momentum is a vector that defines the direction and amount of angular motion of an object. It changes in the presence of an external torque, but is conserved in the absence of external torques. Quantitatively, the angular momentum is defined as the product of the mass (m), the angular velocity (ω), and the radius (r) squared:

$$\mathbf{J} = m\,\omega r^2 \tag{3.4}$$

Since angular momentum is also a vector, its direction is defined by the right-hand rule based on the direction of rotation. The vectors defining the current flow and rotation have the same direction, so there should exist a scalar factor between the magnetic moment and angular momentum. This scalar factor is denoted as γ, such that:

$$\boldsymbol{\mu} = \gamma \mathbf{J} \tag{3.5}$$

It is important to recognize that Equation 3.5 merely states that the magnetic moment (from the rotating charge of the proton) and the angular momentum (from the rotating mass of the proton) have the same direction, with one larger than the other by an unknown factor γ.

To understand what γ represents, let's consider the simplest atomic nucleus, a single proton. First we must make some assumptions, namely that the charge (q) of the proton is an infinitely small point source, the proton rotates about a radius (r), and its rotation has a period (T). From Equation 3.3, we know that the amount of magnetic moment of a moving charge is given by multiplying two properties, the size of the current and the area of the loop it traverses. The former is simply the charge of the proton divided by the time it takes to move around the loop, while the latter is the area of the circle given by its radius r, or πr^2:

$$\mu = IA = \frac{q}{T}\pi r^2 \tag{3.6}$$

We also know that the angular velocity of the proton is equal to the radians of a circle (2π) divided by the time it takes to go around the circle (T). Substituting these values into Equation 3.4, we get the equation for the amount of J:

$$J = m\omega r^2 = m\frac{2\pi r^2}{T} \tag{3.7}$$

Substituting Equations 3.6 and 3.7 into Equation 3.5, it can be derived that:

$$\gamma = \frac{\mu}{J} = \frac{\frac{q}{T}\pi r^2}{m\frac{2\pi r^2}{T}} = \frac{q\pi r^2}{2m\pi r^2} = \frac{q}{2m} \tag{3.8}$$

The final form of this equation is elegant in its simplicity. What Equation 3.8 demonstrates is that the scaling factor (γ) depends only on the charge (q) and mass (m) of the proton. Since the charge and mass of the proton (or any other atomic nucleus) never change, the scaling factor (γ) is a constant for a given nucleus, regardless of the magnetic field strength, temperature, or any other factor. The constant (γ) is known as the gyromagnetic ratio, and is critical for MRI.

In reality, the magnetic moment and angular momentum of a proton cannot be modeled by assuming a simple point charge engaged in circular motion. A proton has a mass of about 1.67×10^{-27} kg and a charge of about 1.60×10^{-19} coulombs so the estimated value of γ is 4.79×10^7 radian/T. However, researchers have determined by experimentation that the real value of γ for a proton is closer to 2.67×10^8 radian/T. Nevertheless, while Equation 3.8 provides only a very rough estimate of the gyromagnetic ratio, it does demonstrate that γ is a unique quantity for a given nucleus. The gyromagnetic ratio has also been measured for other common nuclei; it is 6.73×10^7 for ^{13}C, 2.52×10^8 for ^{19}F, 7.08×10^7 for ^{23}Na, and 1.08×10^8 for ^{31}P (all units in radians per Tesla).

gyromagnetic ratio (γ) The ratio between the charge and mass of a spin. The gyromagnetic ratio is a constant for a given type of nucleus.

parallel state The low-energy state in which an atomic spin precesses around an axis that is parallel to that of the main magnetic field.

antiparallel state The high-energy state in which an atomic spin precesses around an axis that is antiparallel (i.e., opposite) to that of the main magnetic field.

Spins in an External Magnetic Field

If a uniform external magnetic field is applied to proton spins (Figure 3.6), those protons will assume one of the two equilibrium positions: the parallel state (aligned with the magnetic field) or the antiparallel state (opposite to the magnetic field). In MRI, the convention is to refer to the direction along the main magnetic field $\mathbf{B}_0$ as the parallel state. Both states are at equilibrium, although the energy level of the spin in the parallel state is lower than its energy level in the antiparallel state, and hence the spin is more stable in the parallel state. Because the proton spin possesses both a magnetic moment and an angular momentum, it will wobble (or precess) about the direction of the external magnetic field, whether it is in the parallel or the antiparallel state.

Spin precession

Before we examine the effect of an external magnetic field on spin motion in any more detail, let's consider a simple analogy. If we place a magnetic bar that is not spinning into a static magnetic field at an angle θ, it will oscillate back and forth across the main field (Figure 3.14A). However, if the magnetic bar is spinning about its axis, it will wobble around this field instead of oscillating back and forth. This is what happens to atomic nuclei. Because a proton has both magnetic moment and angular momentum, it will wobble around the direction of the external magnetic field (Figure 3.14B). This motion, which we described earlier in the chapter, is known as precession. It is useful to determine the frequency of such precession for any spin used in MRI. In this section, we will derive the precession frequency for hydrogen nuclei in a magnetic field.

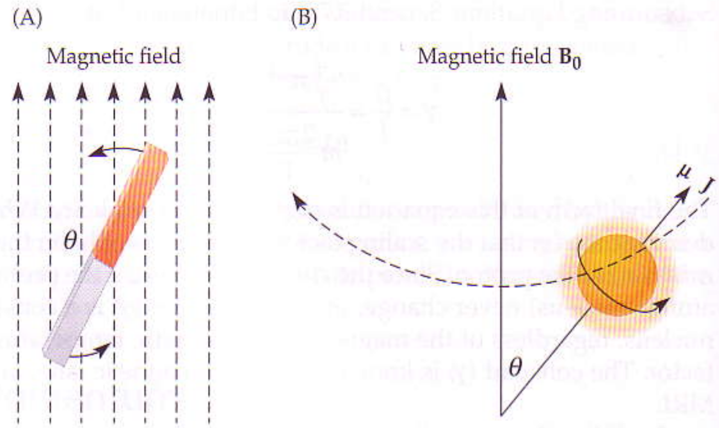


Figure 3.14 Movement within a magnetic field. (A) If a magnetic bar is placed in an external magnetic field, it will oscillate back and forth across the main axis of the field. (B) A spin within an external magnetic field ($\mathbf{B}_0$) has a magnetic moment (μ) and angular momentum (J) and thus will precess around the magnetic field. The cross product of μ and B_0 determines the precession direction. The symbol θ indicates the angle between the axis of spin and the direction of the external field.

A moving charge experiences maximum torque (τ_{max}) equal to the product of its magnetic moment (μ) and field strength (B), when its motion is perpendicular to the main magnetic field:

$$\tau_{\max} = \mu B \tag{3.9}$$

However, if the moving charge is not perpendicular to the main magnetic field, but at some angle θ, then the amount of torque on that charge will be lessened. Specifically, only the component of the magnetic moment vector that is perpendicular to the static field contributes to the torque. Consistent with Figure 3.14B, the perpendicular component of the magnetic moment vector $\boldsymbol{\mu}$ is just $\mu \sin\theta$, since in a right-angled triangle, the sine of a given angle represents the length of the opposite side (the perpendicular component) divided by the length of the hypotenuse (the vector):

$$\tau = \mu B \sin\theta \tag{3.10a}$$

or, in vector form:

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} \tag{3.10b}$$

As we use $\mathbf{B}_0$ to denote the main external magnetic field used for MRI, we get:

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}_0 \tag{3.10c}$$

This means that the torque on the spin's magnetic moment is given by the cross product of the magnetic moment and the main field. Recall also that since torque indicates the change in angular momentum over time, it can be defined as the derivative of angular momentum over the derivative of time:

$$\boldsymbol{\tau} = \frac{d\mathbf{J}}{dt} \tag{3.11}$$

where d is the mathematical symbol indicating change such that $d\theta$ represents the change of θ. Replacing τ in Equation 3.10c with its value from Equation 3.11, we obtain the following equation:

$$\frac{d\mathbf{J}}{dt} = \boldsymbol{\mu} \times \mathbf{B}_0 \tag{3.12}$$

From Equation 3.5 we learned that angular momentum **J** is equivalent to μ/γ, where the constant γ is the gyromagnetic ratio. Thus, there must be a similar generalized expression for how the magnetic moment changes under the main magnetic field, $\mathbf{B}_0$:

$$\frac{d\boldsymbol{\mu}}{dt} = \gamma(\boldsymbol{\mu} \times \mathbf{B}_0) \tag{3.13}$$

Equations 3.12 and 3.13 express in mathematical terms that the torque ($\boldsymbol{\mu} \times \mathbf{B}_0$) on a spin induces changes in the angular momentum and the magnetic moment of that spin over time.

Thought Question

Why does the torque on a spin cause precession? More generally, why does a spinning object precess around a central axis?

To solve for the precession frequency, we need to simplify the vector structure of Equation 3.13. We can do this by breaking down the magnetic moment μ, which is a vector, into its scalar components in three spatial dimensions. At time zero, the components along the three directions can be defined as μ_{x0}, μ_{y0}, and μ_{z0}. The total magnetic moment, $\boldsymbol{\mu}(0)$, is simply the sum of the three components. Here, **x**, **y**, and **z** are unit vectors along the three cardinal directions:

$$\boldsymbol{\mu}(0) = \mu_{x0}\mathbf{x} + \mu_{y0}\mathbf{y} + \mu_{z0}\mathbf{z} \tag{3.14}$$

So, we can transform Equation 3.13 into three separate scalar equations, representing three different dimensions:

$$\frac{d\mu_x}{dt} = \gamma\mu_y B_0 \tag{3.15a}$$

$$\frac{d\mu_y}{dt} = -\gamma\mu_x B_0 \tag{3.15b}$$

$$\frac{d\mu_z}{dt} = 0 \tag{3.15c}$$

We do not go through the entire derivation here, but the result can be summarized simply as follows. The change in the x-component of the magnetic moment at any point in time depends on the current y-component value; at extreme y-values, x changes quickly. The change in the y-component over time depends on the x-component in a similar way. The z-component of the magnetic moment never changes. While this set of equations may seem complex, it merely specifies that the magnetic moment will trace a circular path around the z-axis. As we have already learned, this circular motion is known as precession.

Solving the set of differential equations (3.15) is beyond the scope of this introduction and is left as an exercise for the interested student. The solution, given the initial conditions at time zero (i.e., $\mu_{x0}, \mu_{y0}, \mu_{z0}$), is given by the following equation:

$$\boldsymbol{\mu}(t) = (\mu_{x0}\cos\omega t + \mu_{y0}\sin\omega t)\mathbf{x} + (\mu_{y0}\cos\omega t - \mu_{x0}\sin\omega t)\mathbf{y} + \mu_{z0}\mathbf{z} \tag{3.16}$$

Larmor frequency The resonant frequency of a spin within a magnetic field of a given strength. It defines the frequency of electromagnetic radiation needed during excitation to make spins change to a high-energy state, as well as the frequency emitted by spins when they return to the low-energy state.

where **x**, **y**, and **z** are unit vectors along three spatial dimensions. The terms *cos* ωt and *sin* ωt indicate that magnetic moment precesses at angular velocity ω. Importantly, the angular velocity ω is given by γB_0, which is the Larmor frequency.

Energy Difference between Parallel and Antiparallel States

The net magnetization of a spin system is determined by the relative proportions of spins precessing in the parallel and, antiparallel states. Changing the energy states of some of the spins, therefore, will manipulate the net magnetization. To understand how this change can be effected, three concepts must be discussed: the energy difference between spin states, spin excitation, and signal reception.

To flip a spin from a low-energy (parallel) state to a high-energy (antiparallel) state, we must apply energy. The required energy, or work (W), can be calculated by integrating the torque over the 180-degree rotation angle during this flip:

$$W = -\int_0^{\pi} \tau d\theta = -\int_0^{\pi} \mu B sin\theta d\theta = -\mu B_0 \cos\theta \,|_0^{\pi} = 2\mu B_0 \tag{3.17}$$

where d is the mathematical symbol indicating change such that $d\theta$ represents the change of θ. In our analogy of a bar that can rotate around a pivot point, in order to rotate the bar from the hanging position to the balanced position we must exert a force (torque) for the entire rotation angle. Likewise, to change the spin state of a proton, we must apply enough torque to complete the total amount of work W. Note that W depends only on the magnetic moment μ and the magnetic field B_0. So if the field strength increases, more work will be required to change a spin from one state to another.

We can think of W as being equivalent to the energy difference between the states (ΔE). Remember that when a spin changes states it will either emit or absorb energy in the form of an electromagnetic pulse. The frequency v of this electromagnetic pulse is determined by the energy difference between the states, as given by the Bohr relation:

$$\Delta E = hv \tag{3.18}$$

where h is called Planck's constant. Combining Equations 3.17 and 3.18, we obtain:

$$v = \frac{\Delta E}{h} = \frac{2\mu}{h} B_0 \tag{3.19}$$

It was shown experimentally that the longitudinal component (i.e., along the magnetic field) of the angular momentum **J** of a proton is $\hbar/2$, where $\hbar = h/2\pi$. Thus the longitudinal magnetic moment can be calculated using Equation 3.5 to be:

$$\mu = \gamma \frac{\hbar}{2} = \gamma \frac{h}{4\pi} \tag{3.20}$$

Substituting the result of Equation 3.20 into Equation 3.19, we find that the frequency v of the electromagnetic pulse is equal to the gyromagnetic ratio divided by 2π and multiplied by the magnetic field strength:

$$\nu = \frac{2\mu}{h}B_0 = \mu\frac{2B_0}{h} = \gamma\frac{h}{4\pi}\frac{2B_0}{h} = \frac{\gamma}{2\pi}B_0 \qquad (3.21)$$

Let us pause at this point to review what we know so far. First, a nuclear spin can be characterized by its magnetic moment and angular momentum, both of which are expressed as vectors with the same direction. The magnetic moment is larger than the angular momentum vector by a factor γ, which is known as the gyromagnetic ratio. Second, spins in a magnetic field can take one of two possible states, either a low-energy state parallel to the magnetic field or a high-energy state antiparallel to the magnetic field. To change from the low-energy state to the high-energy state, a spin must absorb electromagnetic energy. Conversely, when changing from a high- to a low-energy state, a spin emits electromagnetic energy. Third, Equation 3.21 demonstrates that the frequency of the absorbed or emitted electromagnetic energy depends only on the gyromagnetic ratio of the spin and the magnetic field strength. So, for any atomic nucleus in a magnetic field with a known field strength, we can calculate the frequency of electromagnetic radiation that is needed to make spins change from one state to another. This frequency ν is also quantitatively equal to the Larmor frequency. Note that although the quantities for angular velocity ω and frequency ν are directly related to each other, they are expressed in different units and differ by a constant factor of 2π : ω is measured in radians per second, while ν is measured in Hertz (Hz), or cycles per second.

You should recognize that $\gamma/2\pi$ in Equation 3.21 is a constant for a given nucleus, expressed in units of frequency divided by field strength. For hydrogen, its numerical value is 42.58 MHz/Tesla. So, for common MR scanners with field strengths of 1.5 T, the Larmor frequency for hydrogen is approximately 63.87 MHz. This frequency is within the radiofrequency band of the electromagnetic spectrum. If we place a human brain into a 1.5-T MR scanner and apply electromagnetic energy at 63.87 MHz, some of the hydrogen nuclei within that brain will change from a low-energy state to a high-energy state. This idea, that energy at a given frequency is needed for changing particular nuclei from one state to another, represents the cardinal principle of magnetic resonance.

The correspondence between ω and ν means that a single quantity, the Larmor frequency, governs two aspects of a spin within a magnetic field: the energy that the spin emits or absorbs when changing energy states and the frequency at which it precesses around the axis of the external magnetic field. This correspondence has important consequences for understanding MR signal generation. The change in energy state is a concept from quantum mechanics, in that spins can only take discrete energy levels with a fixed energy difference between them. The frequency of precession is a concept from classical mechanics, in that it describes the motion of a particle (e.g., proton) through space. Yet, because ω and ν represent the same quantity, these two perspectives, quantum and classical mechanics, are unified in describing MR phenomena. This unification allows us to visualize the quantum behavior of spins using classical mechanics, to derive the basic equations for MR signal generation.

Magnetization of a Spin System

We now have a fundamental and quantitative analysis of the spin properties of individual atomic nuclei in an external magnetic field. Using the equations introduced so far, we can characterize the magnetic moment and angular momentum of a spin, quantify how spins change from a high-energy state to a low-energy state, and predict how spins precess through a magnetic field. However, while the properties of individual nuclei were of interest to Rabi and other early MR physicists, fMRI researchers are not interested in the behavior of a single atomic nucleus. Instead, we are interested in the characteristics of bulk matter within the human brain, which consists of many protons with potentially different properties. Again, because the most abundant nuclei in the human body are hydrogen nuclei, principally within water molecules, we will focus the subsequent discussion on hydrogen.

In the absence of a magnetic field, the spin axes of the nuclei in bulk matter are oriented in random directions, so that the net magnetization (i.e., the sum of all individual magnetic moments) is zero. Once the bulk matter is moved into a magnetic field, each magnetic moment must align itself in either the parallel or antiparallel state. We will refer to the parallel state as p and the antiparallel state as a, and denote the probability for a given nucleus to be found in the parallel state as P_p and the probability for it to be found in the antiparallel state as P_a. Each spin must be in one state or the other, with the sum of the probabilities being one. That is,

$$P_p + P_a = 1 \tag{3.22}$$

If the spins are evenly distributed between these two states, such that there are as many parallel spins as antiparallel spins, there will be no net magnetization. Fortunately for MRI, under normal conditions there are more parallel spins (in the more stable low-energy state) than antiparallel spins (in the high-energy state), and thus there will always be a net magnetization. The relative proportion of the two spin states depends on their energy difference (ΔE) and the temperature (T). This proportion can be determined using Boltzmann's constant, k_B (1.3806×10^{-23} J/K), which governs the probabilities of spin distribution under thermal equilibrium.

$$\frac{P_p}{P_a} = e^{\frac{\Delta E}{k_B T}} \tag{3.23}$$

Note that given the very small value of Boltzmann's constant, $\Delta E / k_B T$ will be much less than one under normal conditions. For very small exponents x, the exponential e^x can be approximated by $1 + x$. Thus, Equation 3.23 can be replaced by:

$$\frac{P_p}{P_a} \approx 1 + \frac{\Delta E}{k_B T} \tag{3.24}$$

Equation 3.24 is called the high-temperature approximation. By algebraically solving Equations 3.22 and 3.24, we obtain:

$$P_p - P_a = \frac{1+\frac{\Delta E}{k_B T}}{2+\frac{\Delta E}{k_B T}} - \frac{1}{2+\frac{\Delta E}{k_B T}} = \frac{\frac{\Delta E}{k_B T}}{2+\frac{\Delta E}{k_B T}} \approx \frac{\Delta E}{2k_B T} \tag{3.25}$$

The quantity $P_p - P_a$ indicates how many more spins are parallel to the magnetic field than are antiparallel. Each of these spins contributes a magnetic moment with magnitude μ along the z-direction. Thus, the total magnetic moment, which is called the bulk magnetization or net magnetization, is simply this proportion multiplied by the number of protons per unit volume (n) times the magnetic moment of each spin in the z-direction. The net magnetization is represented by the symbol **M**. (Here **z** is a unit vector in the z-direction.)

$$\mathbf{M} = (P_p - P_a)n\mu_z\mathbf{z} = \frac{\Delta E}{2k_BT}n\mu_z\mathbf{z} \tag{3.26}$$

At room temperature, the proportional difference between the numbers of hydrogen spins in the parallel and antiparallel states is 0.003% per Tesla, which is a very small amount. Note that the net magnetization is parallel to the main field (i.e., the z-direction) and that as long as the temperature remains unchanged it will not vary in amplitude. If the temperature increases, the net magnetization will decrease. Also, and more importantly, since the difference between the energy states, ΔE, increases proportionally with the strength of the main field, the net magnetization is also proportional to the main field strength. This is why using a strong magnetic field increases the amount of MR signal recorded.

While the net magnetization is initially aligned with the main magnetic field, its precession angle is 0° at equilibrium. When tipped away from this starting position by an excitation pulse, the net magnetization will precess around the main axis of the field, just like a single magnetic moment. We can describe the motion of the net magnetization, following an excitation pulse at time point $t = 0$, in three scalar equations as follows:

$$\frac{dM_x}{dt} = \gamma M_y B_0 \tag{3.27a}$$

$$\frac{dM_y}{dt} = -\gamma M_x B_0 \tag{3.27b}$$

$$\frac{dM_z}{dt} = 0 \tag{3.27c}$$

This equation group is nearly identical to Equation group 3.15, with the only difference being that here we are quantifying the net magnetization of a spin system (M) rather than the magnetic moment of a single spin (μ). The solution to this equation group, for net magnetization at a given point in time, is similar to that given in Equation 3.16:

$$\mathbf{M}(t) = M_{x0}(\mathbf{x}\cos\omega t - \mathbf{y}\sin\omega t) + M_{y0}(\mathbf{x}\sin\omega t + \mathbf{y}\cos\omega t) + M_{z0}\mathbf{z} \tag{3.28}$$

Here, M_{x0}, M_{y0}, and M_{z0} are initial conditions for the net magnetization. In summary, the net magnetization of bulk matter behaves similarly to the magnetic moment of a single spin, in that it precesses at the Larmor frequency around the axis of the main field. This suggests that we may be able to affect the motion of the net magnetization vector in the same way that we can affect the energy state of a single spin: by applying electromagnetic energy at the Larmor frequency. We demonstrate this in the next section.

Excitation of a Spin System and Signal Reception

Now we are ready to explore the concepts governing MR signal generation. To help visualize and subsequently quantify the generated signal, we will adopt a classical mechanics perspective that builds on the equations introduced so far.

Spin excitation

excitation The process of sending electromagnetic energy to a sample at its resonant frequency (also called transmission). The application of an excitation pulse to a spin system causes some of the spins to change from a low-energy state to a high-energy state.

laboratory frame The normal reference frame that is aligned with the magnetic field of the scanner.

By measuring the precession of the net magnetization of a spin system, we can discover some of its properties. For example, based on Equation 3.26 above, we can estimate the number of protons within a unit volume (i.e., proton density) based on the quantity n. But we cannot measure the net magnetization of a spin system directly. Therefore, we need to find an indirect approach that perturbs the spin system away from equilibrium and then measures the response of the system to that perturbation, as in the earlier analogy of lifting an unknown object to estimate its weight. This process is called spin excitation.

In a typical MRI experiment, the object sample to be imaged is placed within a strong, uniform magnetic field at the center of the scanner. We now know that the net magnetization of the sample precesses at the Larmor frequency. For hydrogen, the net magnetization oscillates around the main field vector approximately 64 million times per second, if the magnetic field of the scanner is 1.5 T. Because the magnetization rotates so rapidly, it is extremely difficult to change the magnetization with a single pulse of electromagnetic energy. Instead, energy is applied at a given frequency for an extended period of time. To understand the effect of frequency on an oscillating system, consider the example of a backyard swing set. If you apply energy at the swing's natural frequency by pushing each time the swing is in the same place, even very small pushes will help increase the velocity of the swing, and thereby increase the energy in the system. This phenomenon, where small applications of energy at a particular frequency can induce large changes in a system, is known as resonance. For similar reasons, MRI scanners use specialized radiofrequency coils to transmit an electromagnetic excitation pulse (B_1) at the spin precession (i.e., the Larmor) frequency, in order to exert torque on the spins and perturb them (Figure 3.15).

$$\mathbf{B}_1 = B_1\mathbf{x}\ \cos\omega t - B_1\mathbf{y}\sin\omega t \tag{3.29}$$

We are now ready to examine the effect of this electromagnetic pulse on the sample. Because both the spins and the field generated by the excitation pulse are rotating at the Larmor frequency, we can adopt a reference coordinate system that is also rotating at that frequency. For clarity, we will refer to the normal frame of reference that is aligned with the magnetic field of the scanner as the laboratory frame (Figure 3.16A) and the frame of reference rotat-

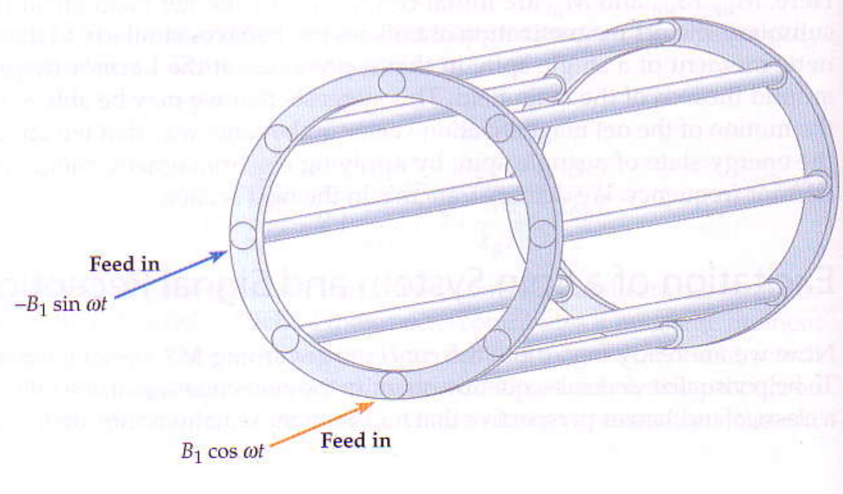


Figure 3.15 Generation of a rotating magnetic field. By driving the volume headcoil using two currents that are out of phase with each other, each fed into a separate part of the circuit, the MR scanner generates a rotating electromagnetic field that allows for efficient excitation at the resonant frequency (i.e., in the radiofrequency range).

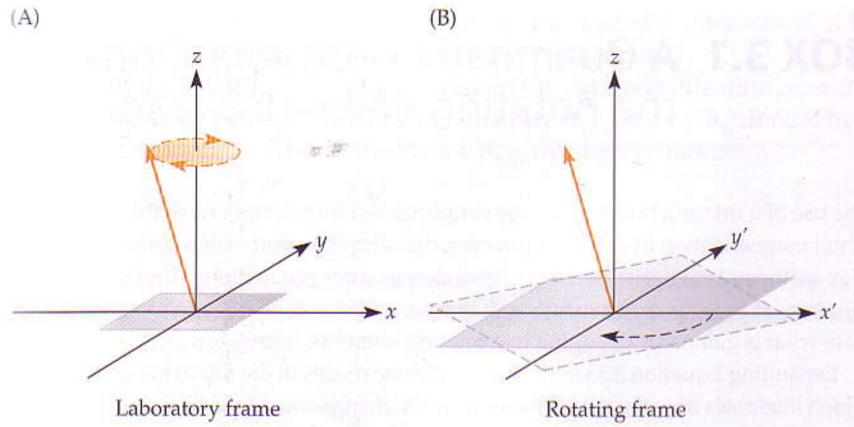


Figure 3.16 Laboratory and rotating reference frames. (A) In the laboratory frame, the magnetization rotates at a given frequency about the main axis (oriented in the z-direction). (B) If a rotating frame ($x' - y'$) is adopted, that spins the transverse plane (the $x - y$ plane) at the same frequency as the magnetization, the magnetization will appear stationary.

ing at the Larmor frequency as the rotating frame (Figure 3.16B). Imagine that you are watching some children ride on a carousel. The laboratory frame is analogous to the situation in which you stand on the ground outside the carousel, watching the children ride around. The rotating frame corresponds to the situation if you get on the carousel and watch the children as you spin around with them. In the latter case, the children will appear stationary, since your rotation speed matches theirs. The unit vectors in the transverse plane within the rotating frame are represented by $\mathbf{x}'$ and $\mathbf{y}'$ and correspond to the following unit vectors in the laboratory frame:

$$\mathbf{x}' = \mathbf{x}\cos\omega t - \mathbf{y}\sin\omega t \tag{3.30}$$

$$\mathbf{y}' = \mathbf{x}\sin\omega t + \mathbf{y}\cos\omega t \tag{3.31}$$

Within the rotating frame, both the spins and the excitation pulse become stationary, making subsequent formulas much simpler. The net magnetization ($\mathbf{M}$) becomes a stationary quantity along the z-direction, while the excitation pulse ($\mathbf{B}_1$) can now be thought of as a stationary vector along the new $\mathbf{x}'$-direction. We therefore would have:

$$\mathbf{M} = M_0\mathbf{z} \tag{3.32a}$$

$$\mathbf{B}_1 = B_1\mathbf{x}' \tag{3.32b}$$

To assess combined magnetization, Equation 3.13 can be rewritten as Equation 3.33, which states that the change in net magnetization over time is the vector product of the net magnetization and the excitation pulse. Similarly to Equation 3.13, this new equation implies that applying a torque ($\gamma\mathbf{M}\times\mathbf{B}$) to the net magnetization will rotate its direction over time:

$$\frac{d\mathbf{M}}{dt} = \gamma\mathbf{M}\times\mathbf{B} \tag{3.33}$$

Note that here the torque on the net magnetization depends on the total magnetic field $\mathbf{B}$ experienced by the spin system. However, this field is the sum of two magnetic fields, the static magnetic field $\mathbf{B}_0$ and the excitation pulse $\mathbf{B}_1$. The effect of the excitation pulse, when it is presented at the resonant frequency of the sample, or on-resonance, is a simple rotation of the net magnetization vector from the z-direction toward the transverse ($x - y$) plane. But if the pulse is

rotating frame A reference frame that rotates at the Larmor frequency of the spin of interest. The rotating frame is adopted to simplify mathematical descriptions of the effects of excitation.

on-resonance excitation The presentation of an excitation pulse at the resonant frequency of the sample, resulting in maximum efficiency.

BOX 3.1 A Quantitative Consideration of the Rotating Reference Frame

The use of a rotating reference frame simplifies our interpretations of the change in net magnetization over time. However, this simplification masks some complex mathematical derivations. Here, we demonstrate the mathematical underpinnings of how net magnetization changes over time, so that readers can appreciate what is gained by changing to a rotating reference frame.

Expanding Equation 3.33 in the rotating frame results in the following equation, which illustrates how the magnetization vector changes over time in each direction:

$$\begin{aligned}\frac{d\mathbf{M}}{dt} &= \frac{d(\mathbf{x}'M_{x'} + \mathbf{y}'M_{y'} + \mathbf{z}'M_{z'})}{dt} \\ &= M_{x'}\frac{d\mathbf{x}'}{dt} + M_{y'}\frac{d\mathbf{y}'}{dt} + \mathbf{x}'\frac{dM_{x'}}{dt} + \mathbf{y}'\frac{dM_{y'}}{dt} + \mathbf{z}'\frac{dM_{z'}}{dt}\end{aligned} \tag{3.34}$$

Since it can be derived that:

$$\frac{d}{dt}\begin{pmatrix}\mathbf{x}'\\ \mathbf{y}'\end{pmatrix} = \begin{pmatrix}\omega\cdot(-\mathbf{x}\sin\omega t - \mathbf{y}\cos\omega t)\\ \omega\cdot(\mathbf{x}\cos\omega t - \mathbf{y}\sin\omega t)\end{pmatrix} = \begin{pmatrix}\omega\cdot(-\mathbf{y}')\\ \omega\cdot(\mathbf{x}')\end{pmatrix} = -\omega\mathbf{z}\times\begin{pmatrix}\mathbf{x}'\\ \mathbf{y}'\end{pmatrix} \tag{3.35}$$

where $\omega\mathbf{z}$ indicates ω as a vector pointing along the z-direction. If we define the changing magnetization in the rotating frame as $\delta\mathbf{M}/\delta t$, in addition to the changing magnetization in the laboratory frame, $d\mathbf{M}/dt$, Equation 3.33 can be rewritten in the following form (Equation 3.36). It indicates that the net magnetization (in the laboratory frame) has two independent components: precession around the z-direction with frequency ω, and a rotation from the longitudinal to the transverse plane (within the rotating frame):

$$\frac{d\mathbf{M}}{dt} = -\omega\mathbf{z}\times\mathbf{M} + \frac{\delta\mathbf{M}}{\delta t} \tag{3.36}$$

By reorganizing Equation 3.36 and substituting Equation 3.33, we can describe a new quantity, $\mathbf{B}_{1\text{eff}}$, which depends on the frequency and amplitude of the applied $\mathbf{B}_1$ field. This quantity is the magnetic field that the spin system actually experiences, and it governs the behavior of the net magnetization within the rotating frame of reference:

$$\begin{aligned}\frac{\delta\mathbf{M}}{\delta t} &= \frac{d\mathbf{M}}{dt} + \omega\mathbf{z}\times\mathbf{M} = \gamma\mathbf{M}\times\mathbf{B} + \omega\mathbf{z}\times\mathbf{M} \\ &= \gamma\mathbf{M}\times(\mathbf{B} - \frac{\omega\mathbf{z}}{\gamma}) = \gamma\mathbf{M}\times\mathbf{B}_{1\text{eff}}\end{aligned} \tag{3.37}$$

presented at a slightly different frequency, so that it is off-resonance, its efficiency greatly decreases. While this loss of efficiency makes intuitive sense, its mathematical derivation is complex. The effective magnetic field experienced by the spin system is not just $\mathbf{B}_1$ but a new field called $B_{1\text{eff}}$ that is influenced by

off-resonance excitation The presentation of an excitation pulse at a frequency other than the resonant frequency of the sample, resulting in reduced efficiency.

$B_{1\text{eff}}$ The effective magnetic field experienced by a spin system during excitation.

both $\mathbf{B}_1$ and $\mathbf{B}_0$. For interested students, we have included the derivation of $\mathbf{B}_{1eff}$ within Box 3.1, and the conclusion is given in Equation 3.37.

Within the rotating frame, the change in the net magnetization over time, notated as $\delta\mathbf{M}/\delta t$, is determined by the quantity $\mathbf{B}_{1eff}$, not by $\mathbf{B}_1$ alone. The net magnetization rotates around the vector $\mathbf{B}_{1eff}$ during excitation:

$$\frac{\delta \mathbf{M}}{\delta t} = \gamma \mathbf{M} \times \mathbf{B}_{1\text{eff}} \tag{3.38}$$

The value of the effective excitation pulse ($\mathbf{B}_{1eff}$) is given by the following equation (derived from Equation 3.37), which has both longitudinal (z) and transverse (x′) components:

$$\mathbf{B}_{1\text{eff}} = \mathbf{z}(B_0 - \frac{\omega}{\gamma}) + \mathbf{x}'B_1 \tag{3.39}$$

If the excitation pulse ($\mathbf{B}_1$) is at the resonant frequency of the spin system, so that $\omega = \gamma B_0$, the term $(B_0 - \omega/\gamma)$ will be equal to zero. This means that if the excitation pulse is on-resonance, the net magnetization vector (in the rotating frame) will simply rotate around the x′-component (Figure 3.17A) with an angular velocity ω_{rot} (Equation 3.40). Note that this equation nicely illustrates why γ is known as the gyromagnetic ratio, in that γ determines the rate at which an introduced magnetic field, in this case $\mathbf{B}_1$, causes a gyroscopic rotation of the net magnetization:

$$\omega_{rot} = \gamma B_{1\text{eff}} = \gamma B_1 \tag{3.40}$$

At this point, we want to pause and emphasize the critical importance of Equation 3.40. This equation tells us that the application of an electromagnetic field at the Larmor frequency will induce a rotation within the rotating frame of reference. This double rotation sounds complicated, but it is actually very simple (see Figure 3.17A). We can think of the rotation within the rotating frame as tipping the net magnetization vector downward from the longitudinal or z-direction into the transverse or $x' - y'$ plane. Within the laboratory frame, the net magnetization vector will follow a spiral path that combines the tipping motion from the rotating frame with precession at the Larmor frequency. This spiral motion in the laboratory frame is known as nutation (Figure 3.17B). The angle θ around which the net magnetization rotates following excitation is determined by the duration T of the applied electromagnetic pulse:

nutation The spiraling change in the precession angle of the net magnetization during an excitation pulse.

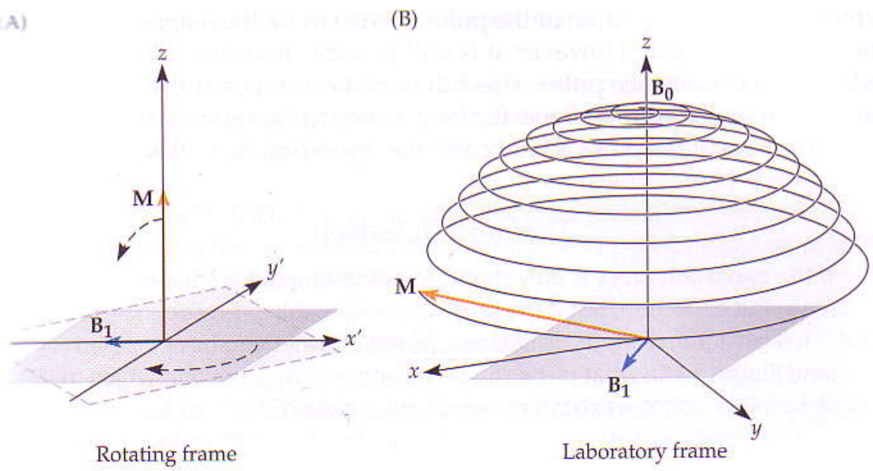


Figure 3.17 Spin nutation. The delivery of an excitation pulse ($\mathbf{B}_1$) causes the longitudinal magnetization (M) to be tipped into the transverse plane. (A) In the rotating frame of reference, whose directions are represented by x' and y', this would look like a simple rotation downward (i.e., the longitudinal magnetization falls smoothly down to the transverse plane). (B) However, in the laboratory frame of reference, the longitudinal magnetization would trace out a complex wobbling path as it rotates downward to the transverse plane. This wobbling motion is known as nutation.

flip angle The change in the precession angle of the net magnetization following excitation.

$$\theta = \gamma B_1 T \tag{3.41}$$

This simple equation determines how long we must apply an electromagnetic field to change the net magnetization vector by an angle θ, called the flip angle. To change the net magnetization by 90 degrees, from along the main field to perpendicular with the main field, the excitation pulse should be presented for a brief period, on the order of milliseconds. Remember that when the net magnetization is entirely in the longitudinal direction, it is stable and does not change over time. Therefore, its amplitude is impossible to measure. But if we tip the net magnetization into the transverse plane with an excitation pulse, there will be large changes in its direction as it rotates. This changing magnetic field can be detected by external receiver coils. In short, by tipping the net magnetization we can create measurable MR signal.

Thought Question

What are the relative proportions of low-energy spins and high-energy spins following application of a 90-degree excitation pulse?

The concept of the $\mathbf{B}_{1\text{eff}}$ (see Equation 3.39) is also very important in understanding off-resonance excitation due to the application of an inhomogeneous field, which results in an actual rotation frequency that does not match the Larmor frequency for a spin system. As such, the difference between B_0 and ω/γ is not zero, which means that the excitation pulse would have a longitudinal component. Because the spins always rotate about the axis of $\mathbf{B}_{1\text{eff}}$, the effectiveness of the excitation pulse will be compromised. To understand the effects of off-resonance excitation, think of an extreme situation in which the excitation pulse has a **z**-component approaching the size of the main field, $\mathbf{B}_0$. As a result, the $\mathbf{B}_{1\text{eff}}$ would be aligned along the z-direction. In this case the $\mathbf{B}_{1\text{eff}}$ would have the same direction as the spins themselves, and would exert no torque on the spins. Thus, their angle of rotation would not change. In mathematical terms, the cross product between two vectors with the same direction is equal to zero. Consequently, an excitation pulse along the same direction as $\mathbf{B}_0$ would have absolutely no effect. If hardware problems with the scanner make the **z**-component of $\mathbf{B}_{1\text{eff}}$ sufficiently large, then full excitation may be impossible to achieve.

But what if $\mathbf{B}_{1\text{eff}}$ is only slightly off-resonance? The rotational trajectories of a perfectly on-resonance pulse and a slightly off-resonance pulse are illustrated in Figure 3.18. The on-resonance pulse has a more efficient trajectory, which reduces the duration of the pulse needed to tip the magnetization into the transverse plane. However, it is still possible to achieve full excitation using the off-resonance pulse. This full excitation comes at the cost of additional time (required to traverse the longer path to the transverse plane), and if the duration of the pulse is held constant, the excitation will be incomplete.

Thought Question

If the excitation pulse is only slightly off-resonance, it is still possible to reach full excitation, but if the pulse is considerably off-resonance, then full excitation cannot be reached. Based on what you have learned (and Figure 3.18), what is the threshold angle of $\mathbf{B}_{1\text{eff}}$ beyond which full excitation cannot be achieved?

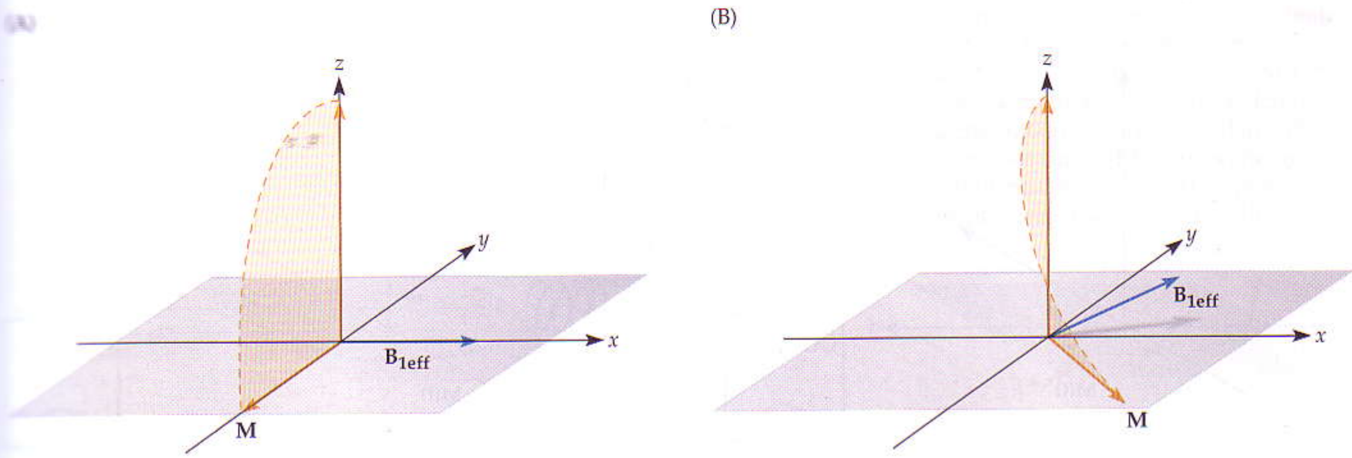


Figure 3.18 On-resonance and off-resonance excitation. (A) Application of an on-resonance excitation pulse will efficiently tip the longitudinal magnetization into the transverse plane (orange trajectory). (B) Application of an off-resonance excitation pulse will result in an inefficient trajectory that takes longer to completely tip the magnetization. The goal of excitation is to achieve full rotation of the magnetization from the longitudinal axis to the transverse plane.

Signal reception

So far, we have shown that an electromagnetic excitation pulse applied by a transmitter coil can change the net magnetization of a spin system. To measure this change, we need another receiver coil (or detector coil). Receiver coils acquire signal through the mechanism of electromagnetic coupling, as governed by Faraday's law of induction. After the magnetization of the sample is tipped to the transverse plane, its precession at the Larmor frequency sweeps across the receiver coil, causing the density of magnetic flux (Φ) experienced by the receiver coil to change over time (Figure 3.19). This change of flux, $d\Phi/dt$, in turn induces an electromotive force (emf) in the coil. By definition,

$$emf = -\frac{d\Phi}{dt} \qquad (3.42)$$

where the magnetic flux penetrating the coil area is given by $\Phi = \int_s B \cdot dS$.

The measurement of electromotive force in a receiver coil is known as reception.

The excitation–reception process simulates the mutual coupling of two coils. Just as a current change in one coil induces a similar current change in another nearby coil through mutual inductance, magnetic field changes in a sample (e.g., the brain) induce magnetic field changes in the receiver coil. The volume magnetic flux generated by the sample and penetrating through the receiver coil can be represented as:

$$\Phi(t) = \int_s \overline{B}_1 \cdot M(t)dv \qquad (3.43)$$

where $\overline{B}_1$ is the magnetic field per unit current of the receiver coil and $M(t)$ is the magnetization created by the sample.

electromotive force (emf) A difference in electrical potential that can be used to drive a current through a circuit. The MR signal is the electromotive force caused by the changing magnetic field across the detector coil.

reception The process of receiving electromagnetic energy emitted by a sample at its resonant frequency (also called detection). As spins return to a low-energy state following the cessation of the excitation pulse, they emit energy that can be measured by a receiver coil.

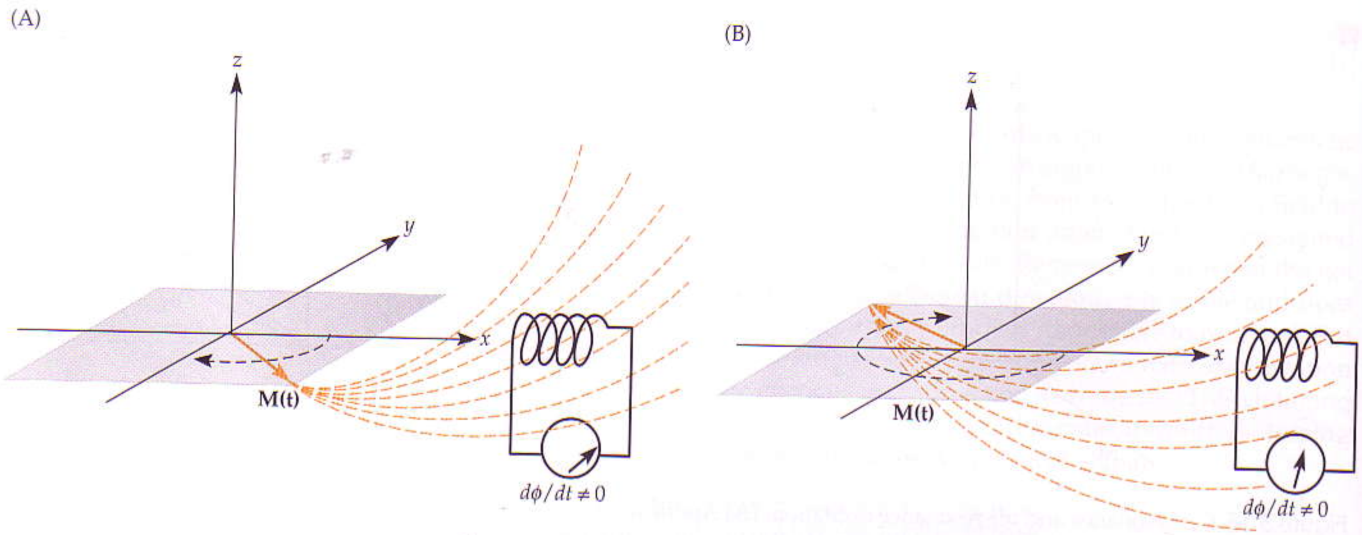


Figure 3.19 MR signal reception. As the net magnetization rotates through the transverse plane, the amount of magnetic flux experienced by the receiver coil changes over time (A and B). The changing flux generates an electromotive force, which provides the basis for the MR signal.

This relationship shows that the magnetic flux through the receiver coil actually depends on the magnetic field that could be produced by the coil. That is, the stronger the magnetic field that can be generated by a coil, the better its reception. Likewise, if a radiofrequency coil can generate a homogeneous magnetic field within a sample, it can also receive signals uniformly from the sample. This relationship is known as the principle of reciprocity.

Substituting Equation 3.43 into Equation 3.42, we get:

$$emf = -i\omega_0 \int_v \overline{B}_1 \cdot M(t)dv \tag{3.44}$$

The additional scaling factor ω_0 comes from taking the time derivative of $M(t)$, which contains the term $\omega_0 t$ (consult Equation 3.28).

Note that the electromotive force oscillates at the Larmor frequency, as do the excitation pulses, so that the receiver coil must be tuned to the resonant frequency to best measure the changes in MR signal. Since both M and ω_0 are proportional to the main field strength B_0, the measured electromotive force is proportional to B_0^2. Stated simply, the amount of MR signal received by the detector coil increases with the square of the magnetic field strength. Unfortunately, the amplitude of noise in the MR signal is proportional to the strength of the magnetic field, so the signal-to-noise ratio increases only linearly with B_0. The effects of field strength on signal and noise will be discussed in detail in Chapter 8.

Equation 3.44 also confirms that before the excitation pulse tips the net magnetization into the transverse plane, there is no detectable electromotive force and thus no MR signal. This is because when the net magnetization is in its original longitudinal direction, its amplitude and direction do not change, so there is no signal to be measured by the receiver coil (Figure 3.20). We emphasize that only changes in the transverse plane contribute to the MR signal.

principle of reciprocity The rule stating that the quality of an electromagnetic coil for transmission is equivalent to its quality for reception (i.e., if it can generate a homogeneous magnetic field at excitation, it can also receive signals uniformly).

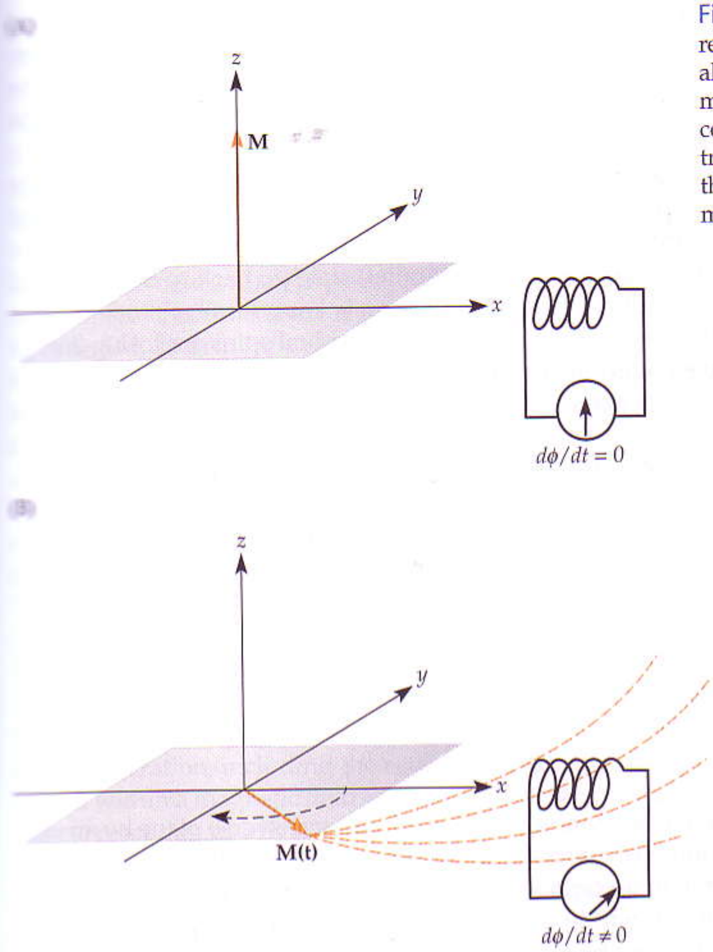


Figure 3.20 Effects of net magnetization orientation on the recorded MR signal. (A) When the net magnetization, **M**, is along the longitudinal axis, there is no detectable change in the magnetic field and thus no electromotive force in the detector coil. (B) After the net magnetization has been tipped into the transverse plane, its motion over time, **M(t)**, causes changes in the measured current within the detector coil. Magnetization must be in the transverse plane for detection using MR.

Relaxation Mechanisms of a Spin System

The MR signal that follows an excitation pulse does not last indefinitely; it decays over time, generally within a few seconds. This phenomenon is called spin relaxation. Two primary mechanisms contribute to the loss of the MR signal: longitudinal relaxation (Figure 3.21A) and transverse relaxation (Figure

relaxation A change in net magnetization over time.

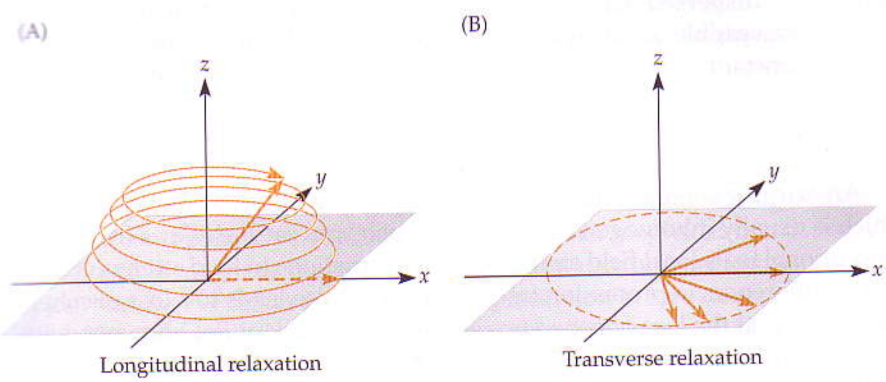


Figure 3.21 T_1 and T_2 relaxation. Schematic illustration of longitudinal relaxation, or T_1 recovery (A), and of transverse relaxation, or T_2 decay (B). The time constant T_1 governs the rate at which longitudinal magnetization recovers, while the time constant T_2 governs the rate at which transverse magnetization decays. Note that T_2* decay is similar to T_2 decay, except that it accounts for both spin–spin interactions (as in T_2) and local field inhomogeneities.

longitudinal relaxation (or spin–lattice relaxation) The recovery of the net magnetization along the longitudinal direction as spins return to the parallel state.

T_1 (recovery) The time constant that describes the recovery of the longitudinal component of net magnetization over time.

transverse relaxation (or spin–spin relaxation) The loss of net magnetization within the transverse plane due to the loss of phase coherence of the spins.

T_2 (decay) The time constant that describes the decay of the transverse component of net magnetization due to accumulated phase differences caused by spin–spin interactions.

3.21B). For a particular substance (e.g., water, fat, or bone) in a magnetic field of a given strength, the rates of longitudinal and transverse relaxation are determined by time constants that we introduce in this section. (We provide the mathematical derivation of these equations and its impact on MR signal in the next two chapters.)

When the excitation pulse is taken away, the spin system gradually loses the energy absorbed during the excitation. The simplest way to think about this energy loss is by using the quantum mechanics perspective. As they lose energy, spins in the high-energy (antiparallel) state go back to their original low-energy (parallel) state. This phenomenon is known as longitudinal relaxation, or spin–lattice relaxation, because the individual spins are losing energy to the surrounding environment, or lattice of nuclei. As increasing numbers of individual spins return to their low-energy state, the net magnetization returns to a direction that is parallel with the main field. From a classical mechanics point of view, the transverse magnetization gradually resumes the longitudinal direction, which it had before the excitation pulse. Because the total magnetization is constant, the growth in the longitudinal magnetization corresponds with a reduction in transverse magnetization and a smaller MR signal. The time constant associated with this longitudinal relaxation process is called T_1, and the relaxation process is called T_1 recovery. The amount of longitudinal magnetization, M_z, present at time t following an excitation pulse is given by Equation 3.45, where M_0 is the original magnetization.

$$\mathbf{M}_z = \mathbf{M}_0(1 - e^{-t/T_1}) \tag{3.45}$$

After the net magnetization is tipped into the transverse plane by an excitation pulse, it is initially coherent, in that all of the spins in the sample are precessing around the main field vector at about the same phase. That is, they begin their precession within the transverse plane at the same starting point. Over time, the coherence between the spins is gradually lost and they fall out of phase. This phenomenon is known as transverse relaxation. In general, there are two main causes for transverse relaxation, one intrinsic and the other extrinsic. The intrinsic cause is from spin–spin interaction: when many spins are excited at once, there is a loss of coherence due to their effects on one another. As an analogy, consider a single racing car driving rapidly around a track; the driver can adopt a high and constant speed because no other cars are present. But in a pack of many cars, the movement of one car influences the speed of the others, making it impossible for all the cars to maintain a constant high speed. Likewise, interactions among spins cause some to precess faster and some slower, causing the relative phases of the precessing spins to become increasingly dispersed over time. The signal loss by this intrinsic mechanism, which is irreversible, is called T_2 decay (Equation 3.46) and is characterized by the time constant T_2.

$$\mathbf{M}_{xy} = \mathbf{M}_0 e^{-t/T_2} \tag{3.46}$$

An extrinsic source of differential spin effects is the external magnetic field, which is usually inhomogeneous. Because each spin precesses at a frequency proportional to its local field strength, spatial variations in field strength cause spatial differences in precession frequencies. This also leads to a loss of coherence. Note that the loss of coherence caused by the lack of field homogeneity can be reversed with specialized pulse sequences, as will be discussed further in Chapter 5. The combined effects of spin–spin interaction and field inhomogeneity lead to signal loss known as T_2* decay, characterized by the time constant T_2*. Note that T_2* decay is always faster than T_2 decay alone, since it includes the additional factor of field inhomogeneity, and thus for any substance the time constant T_2* is always smaller than T_2. The equation for T_2* decay is similar to that for T_2 decay. We will discuss T_2* decay again in Chapters 5 and 7, because it plays a critical role in the BOLD contrast that we use for fMRI.

These relaxation processes constrain how much MR signal can be acquired following a single excitation pulse. Since transverse magnetization decays over a short period of time, there is a limited window within which MRI data can be collected. To acquire a very complex, high-resolution anatomical image, a sample must often receive a sequence of many excitation pulses to allow collection of all data points. The trade-offs between MR signal and acquisition time are discussed further in Chapter 5. It is important to realize that relaxation processes are not a problem for MRI, but instead they provide the capability for measuring different properties of matter. The versatility of MRI as an imaging tool results from its sensitivity to the different relaxation properties of tissues.

T_2* (decay) The time constant that describes the decay of the transverse component of net magnetization due to both accumulated phase differences and local magnetic field inhomogeneities. T_2* is always shorter than T_2. BOLD-contrast fMRI relies on T_2* contrast.

Bloch equation An equation that describes how the net magnetization of a spin system changes over time in the presence of a time-varying magnetic field.

The Bloch Equation for MR signal generation

The physical principles introduced in this chapter provide an overview of MR signal generation, including the establishment of net magnetization of a spin system within a magnetic field, excitation of those spins using electromagnetic pulses, reception of MR signal in detector coils, and relaxation of magnetization over time. Because these components are related, we can describe MR phenomena in a single equation, which is a modification of Equation 3.33 that includes the T_1 and T_2 effects:

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B} + \frac{1}{T_1}(\mathbf{M}_0 - \mathbf{M}_z) - \frac{1}{T_2}(\mathbf{M}_x + \mathbf{M}_y) \tag{3.47}$$

Stated generally, the net magnetization vector of a spin system precesses around the main magnetic field axis at the Larmor frequency, with its change in the longitudinal or z-direction governed by T_1 and its change in the transverse plane governed by T_2. This equation is called the Bloch equation, after the physicist Felix Bloch (see Chapter 1), and it describes the behavior of the net magnetization of a spin system in the presence of a magnetic field that varies over time. As we will learn in the following chapters, solutions to this equation provide mathematical representations of magnetization during the steady state, excitation, and relaxation. Thus, the Bloch equation provides the theoretical foundation for all MRI experiments.

Summary

A set of physical principles underlies the generation of the MR signal. The primary concepts are those of nuclear spin and net magnetization. Atomic nuclei with a magnetic moment and angular momentum are known as spins, and they exhibit rapid gyroscopic precession in an external magnetic field. The axis around which they precess is known as the longitudinal direction, and the plane in which they precess is known as the transverse plane. Each spin adopts either a low- or a high-energy state. These low- and high-

REFER TO THE
fMRI
COMPANION WEBSITE AT
www.sinauer.com/fmri2e
for study questions and Web links.

energy states are parallel and antiparallel to the magnetic field, respectively. Under normal conditions, the net magnetization from all spins is a vector parallel to the static magnetic field. By applying an electromagnetic pulse that oscillates at the resonant (Larmor) frequency of the spins, in a process known as excitation, one can tip the net magnetization vector from the longitudinal direction into the transverse plane. This causes the net magnetization to change over time in the transverse plane, generating the MR signal that can be measured using an external detector coil. A single formula, known as the Bloch equation, forms the basis for the quantitative description of magnetic resonance phenomena.

Suggested Readings

Haacke, E. M., Brown, R. W., Thompson, M. R., and Venkatesan, R. (1999). *Magnetic Resonance Imaging: Physical Principles and Sequence Design.* John Wiley & Sons, New York. *A comprehensive encyclopedia of the theoretical principles of MRI.*

Jin, J. (1999). *Electromagnetic Analysis and Design in Magnetic Resonance Imaging.* CRC Press, Boca Raton, FL. *This book describes the basic theory and design underlying the radiofrequency hardware for MR signal excitation and reception.*

Slichter, C. P. (1996). *Principles of Magnetic Resonance* (3rd ed.). Springer-Verlag, New York. *This book provides a detailed mathematical treatment of the physics of MRI.*